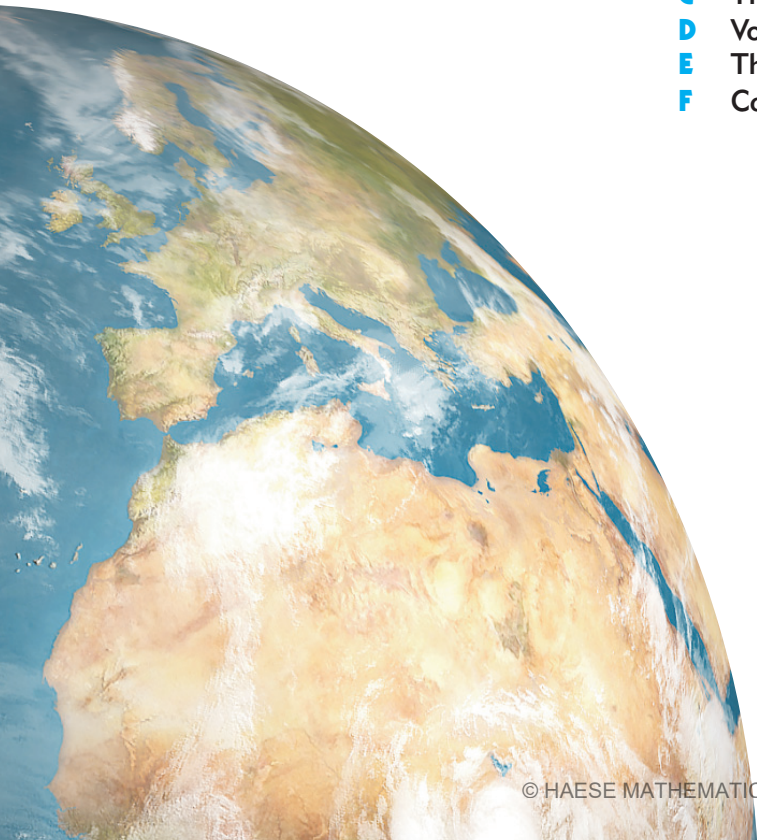


Chapter 10

Measurement: Area, volume, and capacity

Contents:

- A** Area
- B** The area of a rectangle
- C** The area of a triangle
- D** Volume
- E** The volume of a rectangular prism
- F** Capacity

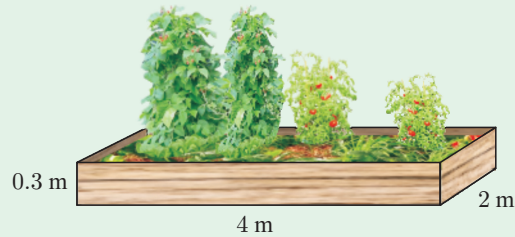


OPENING PROBLEM

Peter is building a raised garden bed for his son's primary school. The garden bed will be 4 m long, 2 m wide, and 0.3 m high.

Things to think about:

- What *area* of weed mat is needed to cover the base of the garden bed?
- What *volume* of soil is needed to fill the garden bed?
- Peter buys soil in bags with *capacity* 20 litres. How many bags of soil will Peter need to fill the garden bed?



A

AREA

When we talk about how large a shape is, we are usually interested in how much *surface* it covers.

Area is a measurement of the size of a surface.

DISCUSSION

- Why might you want to measure the area of:
 - ▶ a classroom floor
 - ▶ the walls of a room?
- What areas do these people need to measure?
 - ▶ a tiler
 - ▶ a dressmaker
 - ▶ a glazier

ACTIVITY 1

SQUARE UNITS

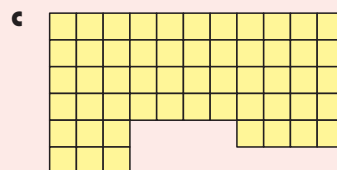
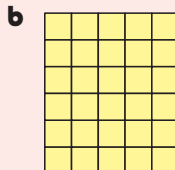
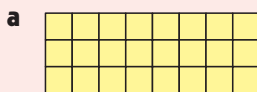
One way to measure an area is to cover it with identical shapes and then count them.

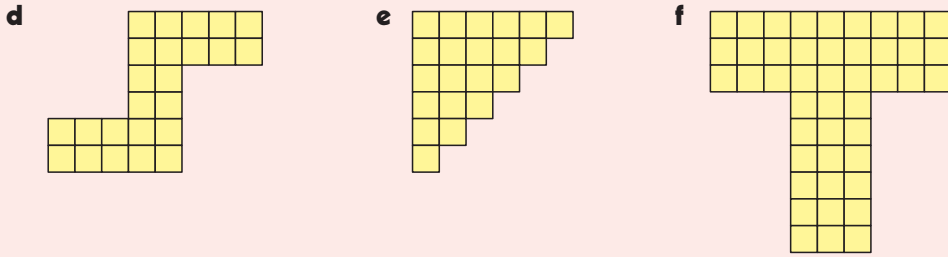
Since it is easy to place identical squares together with no gaps between them, a **square unit** can be used to measure area. We write a square unit as 1 unit^2 .



What to do:

- Find the area of each shape in square units:





- 2** Which of the shapes in **1** has the:
- a** largest area **b** smallest area?
- 3** Use 20 square units to construct:
- a** a rectangle **b** a hexagon **c** a decagon
- d** a polygon with as many sides as possible.
- 4** **a** What other *regular* shapes can be placed together with no gaps between them?
- b** Why do you think the square was chosen to be the unit for measuring area?

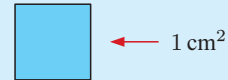
In **Chapter 9** we used **millimetres**, **centimetres**, **metres**, and **kilometres** to measure length.

We use these units of length as a basis for commonly used square units of area.

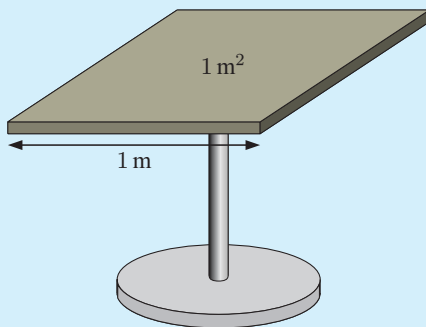
1 square millimetre (mm^2) is the area enclosed by a square of side length 1 mm.



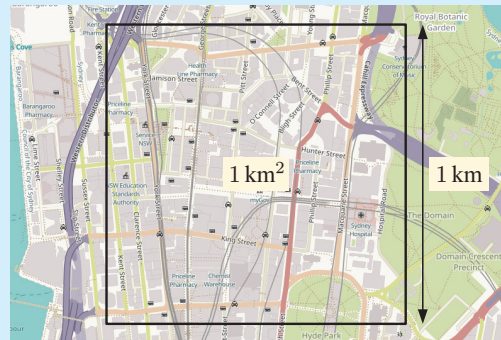
1 square centimetre (cm^2) is the area enclosed by a square of side length 1 cm.



1 square metre (m^2) is the area enclosed by a square of side length 1 m.



1 square kilometre (km^2) is the area enclosed by a square of side length 1 km.



© OpenStreetMap contributors

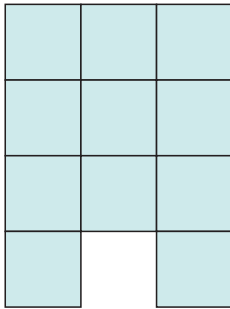
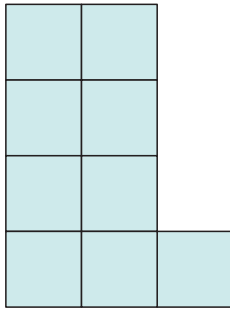
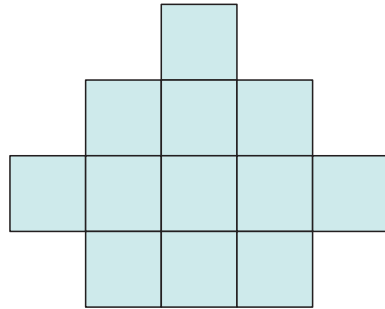
EXERCISE 10A

1 Match each description with the correct area:

- a** a picnic rug
- b** a sports stadium
- c** New Zealand
- d** a drink coaster
- e** this dot ●
- f** a golf course
- g** a suburban park
- h** a coin

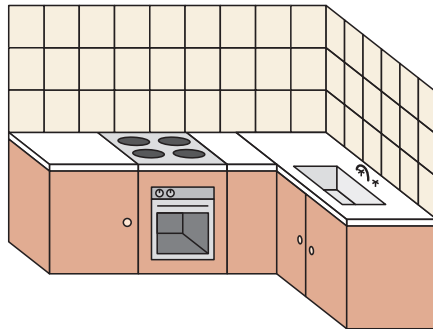
- A** 0.8 km^2
- B** 100 cm^2
- C** 5 m^2
- D** 7.5 cm^2
- E** 600 m^2
- F** 7 mm^2
- G** $268\,000 \text{ km}^2$
- H** $20\,000 \text{ m}^2$

2 Each of the squares in these shapes has area 1 square centimetre. Count the squares to find the area of each shape.

a**b****c**

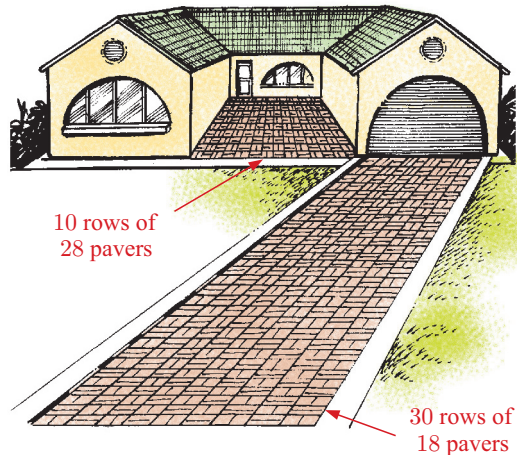
3 The diagram shows Christine's kitchen.

- a** How many tiles have been used?
- b** 20 tiles cover an area of one square metre. How many square metres of tiles are there in Christine's kitchen?
- c** The tiles cost \$44 per square metre. Find the total cost of the tiles.



4 Look at the picture. The brick pavers in the courtyard are identical to those in the driveway.

- a** How many pavers have been used for:
 - i** the driveway **ii** the courtyard?
- b** 50 pavers cover an area of one square metre. How many square metres of paving are there in total?
- c** The pavers cost \$32 per m^2 , and the cost of laying them was \$28 per m^2 . Find the total cost of the paving.



- 5 Each of the small squares in these shapes has area 1 square millimetre. Find the area of each shape.

a



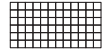
b



c



d



DISCUSSION

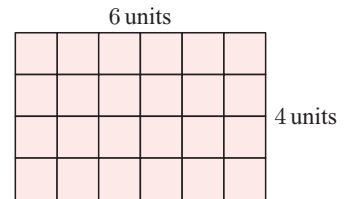
When you found the areas of the shapes in question 5, did you count each individual square? Is there a quicker way to find the area of each shape?

B

THE AREA OF A RECTANGLE

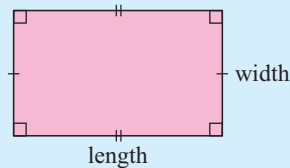
Consider a rectangle 6 units long and 4 units wide.

The area of this rectangle is 24 units^2 , and we can find this by multiplying $6 \times 4 = 24$.



In general, the area of a rectangle is given by the rule:

Area of rectangle = length $\times$ width

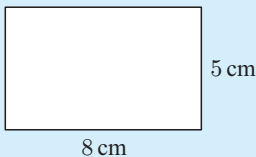


Example 1

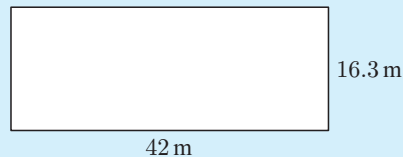
Self Tutor

Find the area of each rectangle:

a



b



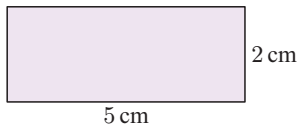
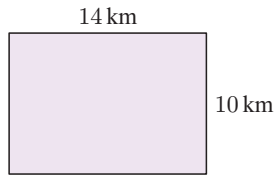
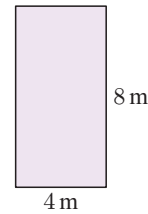
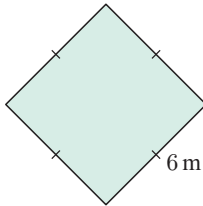
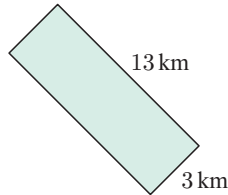
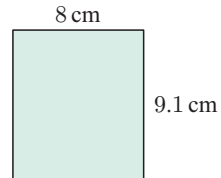
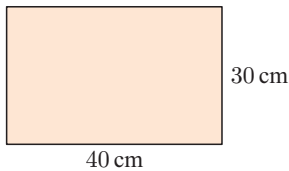
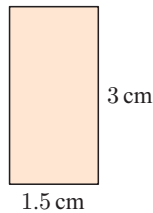
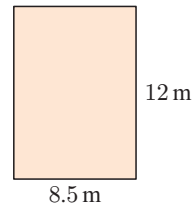
$$\begin{aligned} \text{a Area} &= \text{length} \times \text{width} \\ &= 8 \text{ cm} \times 5 \text{ cm} \\ &= 40 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{b Area} &= \text{length} \times \text{width} \\ &= 42 \text{ m} \times 16.3 \text{ m} \\ &= 42 \text{ m} \times \frac{163}{10} \text{ m} \\ &= \frac{42 \times 163}{10} \text{ m}^2 \\ &= \frac{6846}{10} \text{ m}^2 \\ &= 684.6 \text{ m}^2 \end{aligned}$$

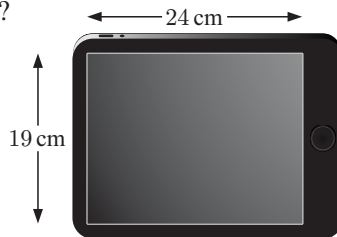
$$\begin{array}{r} 1 \ 6 \ 3 \\ \times \quad 4 \ 2 \\ \hline 126 \\ + 2520 \\ \hline 6846 \end{array}$$

EXERCISE 10B

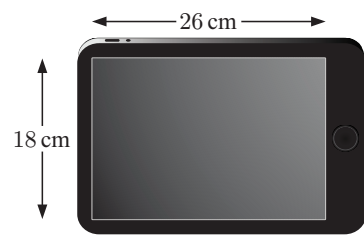
- 1** Find the area of each rectangle. Make sure you answer with the correct units.

a**b****c****d****e****f****g****h****i**

- 2** Whose tablet screen is larger?

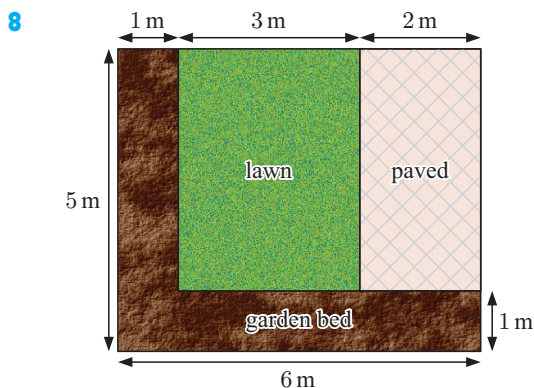


Meredith's tablet



Gemma's tablet

- 3** Tom wants to cover his 12 m by 10 m backyard with lawn. Seed for the lawn costs \$8 per square metre. Find:
- a** the area of the lawn
 - b** the total cost of the seed for the lawn.
- 4** A 6 m by 7.5 m ceiling is to be painted. One litre of paint covers 15 square metres. Find:
- a** the area of the ceiling
 - b** the total amount of paint required.
- 5** A rectangular swimming pool is 8 m long and 4.6 m wide. Find:
- a** the perimeter of the pool
 - b** the area of the pool.
- 6** An A4 sheet of paper is 29.7 cm by 21.0 cm. Find the area of an A4 sheet of paper.
- 7** A 4.8 m by 6 m room has a 2 m by 2.8 m rug on the floor. Find the area of exposed floor.



A backyard has the design shown.

Find the area of the:

- a whole backyard
- b paved section
- c lawn
- d garden bed.

- 9 The base of Ted's shower is 1.2 m long and 90 cm wide. It is to be covered with tiles which are 6 cm long and 5 cm wide.

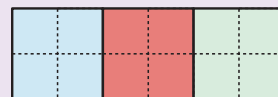
- a Find the area of the shower base in square centimetres.
- b Find the area of each tile.
- c How many tiles are required?

When finding areas, each measurement must be in the same units.

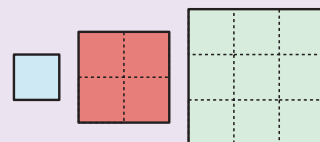


PUZZLE

It is easy to put squares of *equal size* together to form a rectangle. For example, in this diagram three 2×2 squares are put together to form a 6×2 rectangle.



It is much harder to form a rectangle if the squares have different sizes! For example, it is impossible to form a rectangle using the 1×1 square, 2×2 square, and 3×3 square opposite.



In 1925, Polish mathematician **Zbigniew Moron** discovered that one of each of the squares 1×1 , 4×4 , 7×7 , 8×8 , 9×9 , 10×10 , 14×14 , 15×15 , and 18×18 can be used to form a rectangle.

What to do:

- 1 Use grid paper to draw squares with the sizes given above, or click on the icon and print the squares.
- 2 Cut out the squares, and arrange them to form a rectangle.

Hint: The total area of the squares can help you work out the dimensions of the rectangle.

SQUARES



C

THE AREA OF A TRIANGLE

INVESTIGATION 1

THE AREA OF A TRIANGLE

You will need: scissors, ruler, pencil, graph paper.

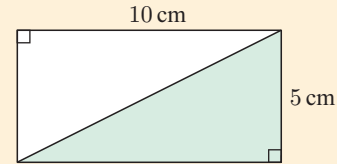
GRAPH PAPER

What to do:

- 1
 - a Draw a large rectangle using the grid on the graph paper.
 - b Draw one diagonal, then colour one of the triangles green.
 - c Cut out the two triangles, then arrange them on top of each other so you can see they have the same shape.
 - d Copy and complete:

The areas of the two triangles are

The area of each triangle is the area of the rectangle.



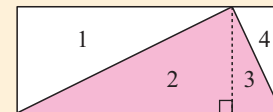
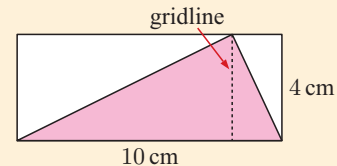
- 2
 - a Draw another large rectangle using the graph paper.
 - b Select a point where the top boundary meets a vertical grid line. Use it to draw triangles as shown.
 - c Colour in the pink triangle.
 - d Now divide the pink triangle along the grid line so you form four regions in total.
 - e Using what you found in 1, copy and complete:

The areas of regions 1 and 2 are

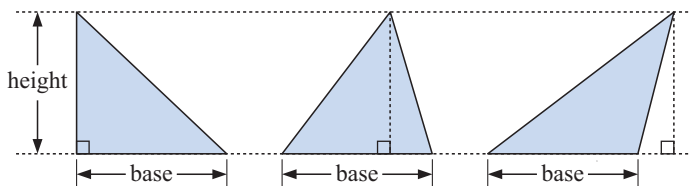
The areas of regions 3 and 4 are

So, area 2 + area 3 = area 1 + area 4.

The total area of the pink triangle is the area of the rectangle.



From the **Investigation** you should have found that the area of a triangle is half the area of a rectangle which has the *same base and height* as the triangle.



$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

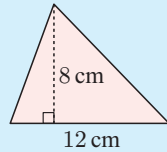
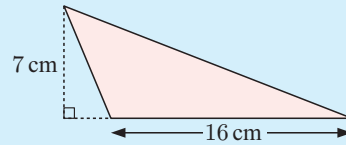
Even though they have different shapes, these triangles have the same base and height. They therefore have the same area.

DEMO



Example 2

Find the area of each triangle:

a**b****a** Area of triangle

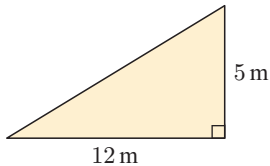
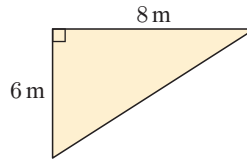
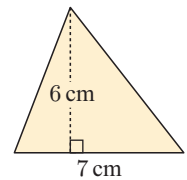
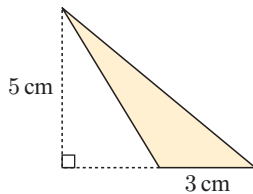
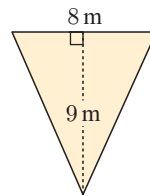
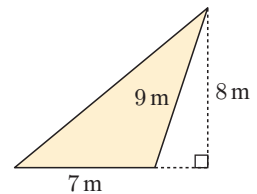
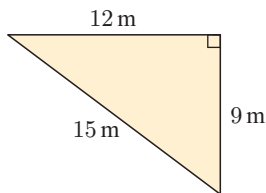
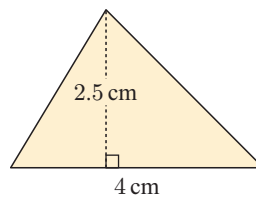
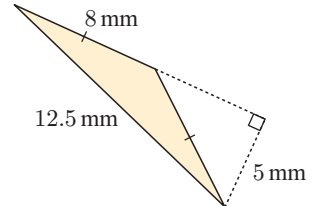
$$\begin{aligned}
 &= \frac{1}{2} \times \text{base} \times \text{height} \\
 &= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm} \\
 &= 48 \text{ cm}^2
 \end{aligned}$$

b Area of triangle

$$\begin{aligned}
 &= \frac{1}{2} \times \text{base} \times \text{height} \\
 &= \frac{1}{2} \times 16 \text{ cm} \times 7 \text{ cm} \\
 &= 56 \text{ cm}^2
 \end{aligned}$$

EXERCISE 10C

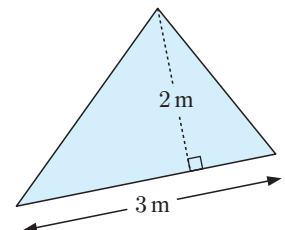
1 Find the area of each triangle:

a**b****c****d****e****f****g****h****i**

2 Alice is buying five identical shade sails with the dimensions shown. The shade cloth costs \$28 per m².

a Find the total area of shade cloth.

b Find the total cost of the shade cloth.

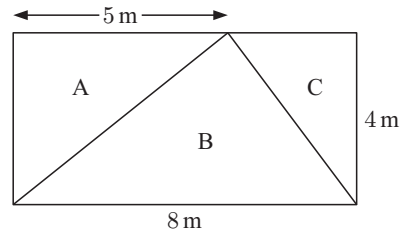


3 A rectangular tarpaulin is cut into three triangles.

a Find the area of:

- i the original rectangle ii triangle A
- iii triangle B iv triangle C.

b Check that the sum of the areas of the triangles is equal to the area of the original rectangle.

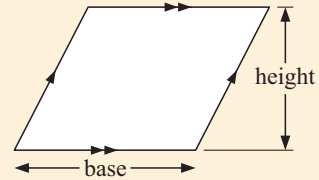


INVESTIGATION 2

THE AREA OF A PARALLELOGRAM

Look at the parallelogram alongside.

How can we describe the area of the parallelogram in terms of its base and its height?

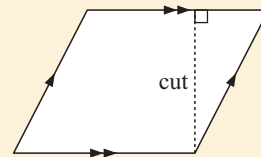


What to do:

1 a Using your ruler, carefully draw a large parallelogram on a piece of paper. It is important that the sides are parallel.

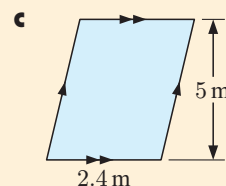
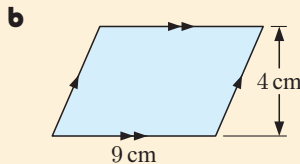
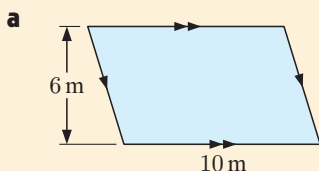
b Cut out a triangle from one end of the parallelogram as shown.

Use the two pieces to form a rectangle.



c Hence explain why the area of the parallelogram is equal to the length of its base times its height.

2 Find the area of each parallelogram:



D

VOLUME

The stone occupies more space than the pebble.

We say that the stone has greater *volume* than the pebble.



stone



pebble

The **volume** of a solid is the amount of space it occupies.