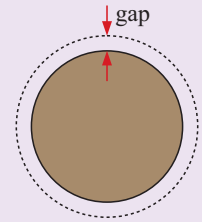
PUZZLE ROPE AROUND THE EARTH

Consider these two scenarios:

1 A loop of rope is placed tightly around a circular table.
1 metre of rope is then added to the loop, and the rope is pulled into a circle so there is an equal gap between the rope and the table.



2

A loop of rope is placed tightly around the Earth.
Again, 1 metre of rope is then added to the loop, and the rope is pulled into a circle so there is an equal gap between the rope and the Earth.

Do you think the gap will be larger in scenario **1** or scenario **2**?
Perform some calculations to determine whether you are correct!

D AREA

Area is the measurement of the size of a surface.

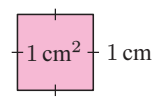
We can measure area using the following units:

1 square millimetre (mm^2) is the area enclosed by a square of side length 1 mm.
1 square centimetre (cm^2) is the area enclosed by a square of side length 1 cm.
1 square metre (m^2) is the area enclosed by a square of side length 1 m.
1 hectare (ha) is the area enclosed by a square of side length 100 m.
1 square kilometre (km^2) is the area enclosed by a square of side length 1 km.

AREA CONVERSIONS

We can use the length unit conversions to help us convert from one area unit to another.

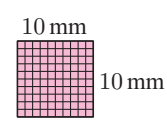
For example, a square with sides of length 1 cm has area 1 cm^2:



We could also measure the sides of this square in millimetres:

Each of the small squares has area 1 mm^2.

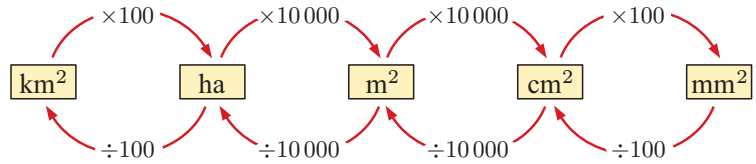
There are $10 \times 10 = 100$ square millimetres in the square,
so $1\ cm^2 = 100\ mm^2$.



Likewise:

- $1\text{ m}^2 = 1\text{ m} \times 1\text{ m}$
  $= 100\text{ cm} \times 100\text{ cm}$
  $= 10\,000\text{ cm}^2$
- $1\text{ ha} = 100\text{ m} \times 100\text{ m}$
  $= 10\,000\text{ m}^2$
- $1\text{ km}^2 = 1\text{ km} \times 1\text{ km}$
  $= 1000\text{ m} \times 1000\text{ m}$
  $= 1\,000\,000\text{ m}^2$
  $= 100 \times 10\,000\text{ m}^2$
  $= 100\text{ ha}$

$1\text{ km}^2 = 100\text{ ha}$
$1\text{ ha} = 10\,000\text{ m}^2$
$1\text{ m}^2 = 10\,000\text{ cm}^2$
$1\text{ cm}^2 = 100\text{ mm}^2$



Example 6 — Self Tutor

Convert:

a 2.2 ha into m^2 **b** 540 mm^2 into cm^2.

a
$= 2.2 \times 10\,000\text{ m}^2$
$= 22\,000\text{ m}^2$

b
$= 540 \div 100\text{ cm}^2$
$= 5.4\text{ cm}^2$

EXERCISE 11D

1 Choose the correct answer:

a The area of a postage stamp is about:

A 10 mm^2 **B** 10 cm^2 **C** 10 m^2 **D** 10 ha

b The area of the floor of an elevator is about:

A 3 mm^2 **B** 3 cm^2 **C** 3 m^2 **D** 3 ha

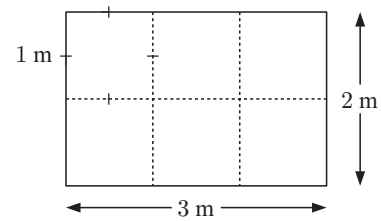
c The area of a school is about:

A 2 cm^2 **B** 2 m^2 **C** 2 ha **D** 2 km^2

2 Erin has a rectangular vegetable garden measuring 2 m by 3 m.

a Suppose the garden is divided into squares with side length 1 m.

State the area of the garden in m^2.



b Now suppose the garden is divided into squares with side length 1 cm.

i State the number of rows and columns of squares.

ii Hence find the area of the garden in cm^2.

3 Convert:

a 5 cm^2 into mm^2 **b** 2500 mm^2 into cm^2 **c** 7 ha into m^2

d 3.6 m^2 into cm^2 **e** 0.4 km^2 into ha **f** 83 cm^2 into mm^2

g 80 ha into m^2 **h** 15 600 cm^2 into m^2 **i** 1200 ha into km^2

j 900 mm^2 into cm^2 **k** 76 000 m^2 into ha **l** 2.8 cm^2 into mm^2

m 0.25 km^2 into ha **n** 12.48 m^2 into cm^2 **o** 0.0092 m^2 into mm^2.

4 A photograph has area 150 cm^2. Express this area in mm^2.

5 Three farmers Alessio, Bruno, and Carlos own blocks of land with the following areas:
Alessio 2.15 km^2, Bruno 320 ha, Carlos 640 000 m^2.
Which farmer owns the:

a largest block **b** smallest block?

6 The gardener of a school has 5.7 ha of lawn to manage. He must apply 60 g of fertiliser to every m^2 of lawn. He wants to buy fertiliser in 25 kg bags, each costing \$19.85.

a How much fertiliser will be required?

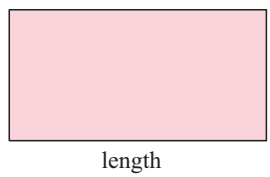
b How much will the gardener spend on fertiliser?

7 One hectare is about 2.471 05 acres. How many square metres are there in an acre?

E AREA FORMULAE

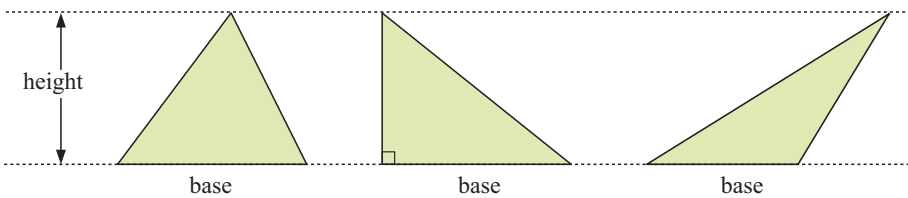
In previous years we have seen formulae for the area of **rectangles**, **triangles**, **parallelograms**, and **trapezia**.

RECTANGLES



Area of rectangle = length × width

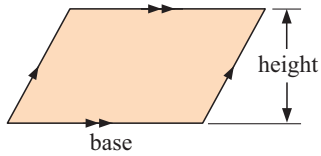
TRIANGLES



DEMO

Area of triangle $= \frac{1}{2} \times$ **base** $\times$ **height**

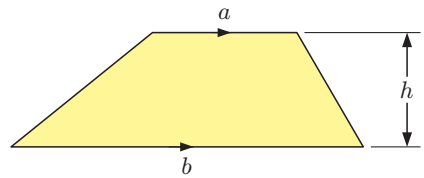
PARALLELOGRAMS



Area of parallelogram = base × height

DEMO

TRAPEZIA



DEMO

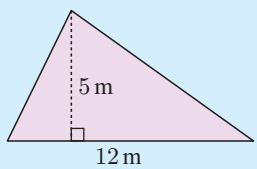
Area of trapezium = average length of parallel sides × distance between parallel sides

$$= \left(\frac{a+b}{2}\right) \times h$$

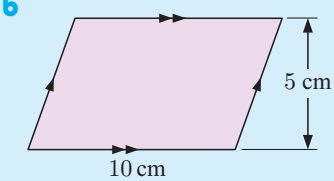
Example 7 Self Tutor

Find the area of:

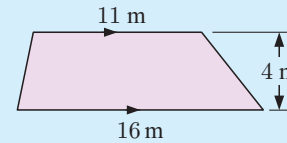
a



b



c



a Area

$= \frac{1}{2} \times$ base $\times$ height

$= \left(\frac{1}{2} \times 12 \times 5\right)$ m^2

$= 30$ m^2

b Area

= base × height

$= (10 \times 5)$ cm^2

$= 50$ cm^2

c Area

$= \left(\frac{a+b}{2}\right) \times h$

$= \left(\frac{11+16}{2}\right) \times 4$ m^2

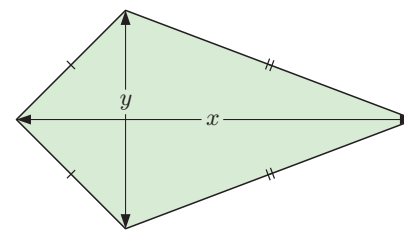
$= 54$ m^2

KITES

Area of kite

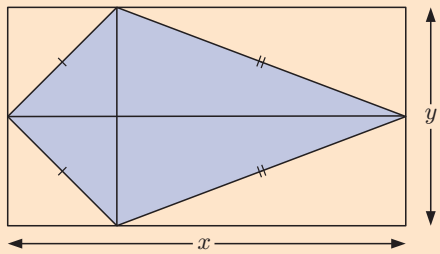
$= \frac{1}{2} \times$ **the product of the lengths of the diagonals**

$= \frac{1}{2}(x \times y)$



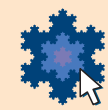
Proof:

Suppose we draw the kite within a rectangle with length x and width y. This creates four smaller rectangles, each divided in half by a side of the kite.



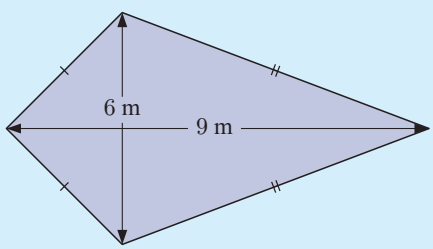
$$\text{Area of kite} = \frac{1}{2}(\text{area of rectangle})$$
$$= \frac{1}{2}(x \times y)$$

DEMO



Example 8 **Self Tutor**

Find the area of the kite:

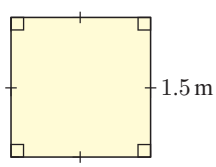


$$\text{Area} = \frac{1}{2}(x \times y)$$
$$= \frac{1}{2}(9 \times 6) \text{ m}^2$$
$$= \frac{1}{2} \times 54 \text{ m}^2$$
$$= 27 \text{ m}^2$$

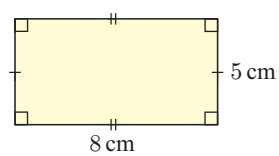
EXERCISE 11E

1 Find the area of each polygon:

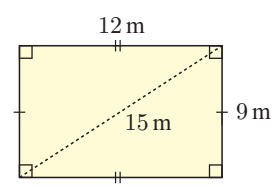
a



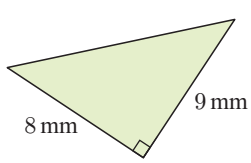
b



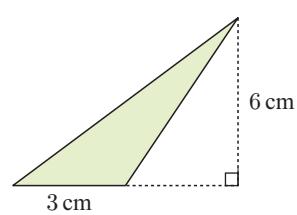
c



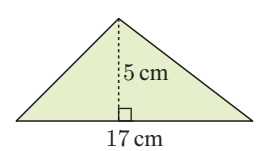
d



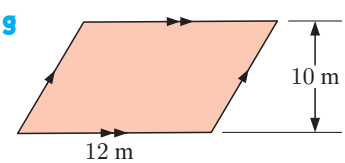
e



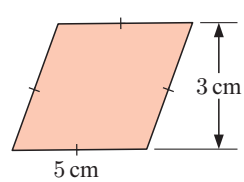
f



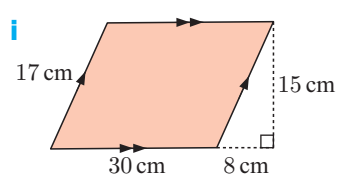
g



h

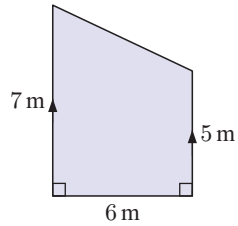


i

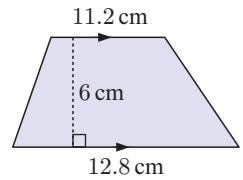


2 Find the area of each polygon:

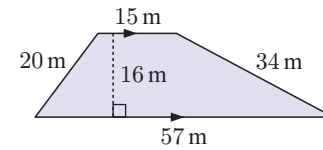
a



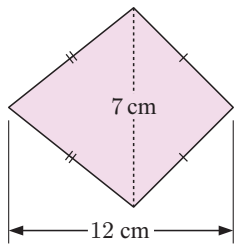
b



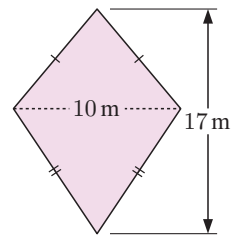
c



d



e



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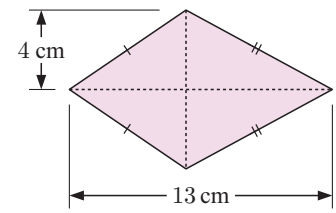
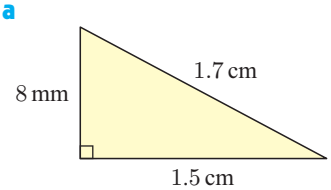
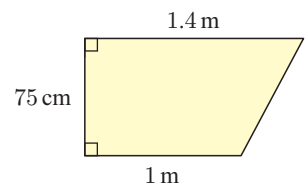


3 Find the area of each polygon:

a



b



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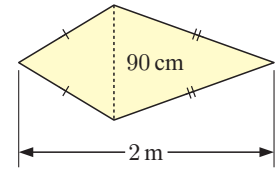


4 A double bed sheet is 254 cm long and 228 cm wide. A queen bed sheet is 274 cm long and 245 cm wide.

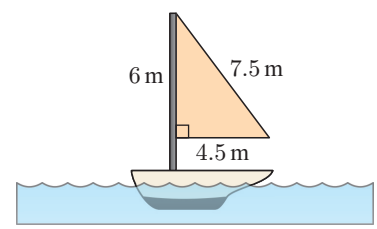
a Find the area of:

i a double bed sheet **ii** a queen bed sheet.

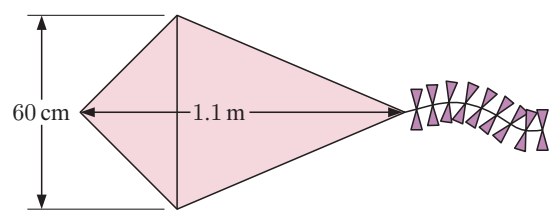
b What percentage larger is the area of a queen bed sheet than the area of a double bed sheet?

5 **a** Find the amount of material required to make the sail shown.

b The material costs $19.50 per square metre. Find the cost of the sail.



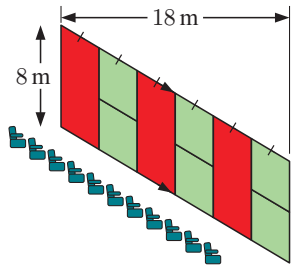
6 Find, in m^2, the total amount of material required to make 30 of these kites:



7 Anya's farm measures 800 metres by 1.2 km. 36 hectares are sown with wheat. Find the percentage of her farm that is sown with wheat.

8 A square paving brick has area 225 cm^2. If each brick fits tightly in place, how many bricks are needed to pave a 1.2 m by 12 m path?

9



A feature wall at a movie theatre is made from green and red panels, as shown.

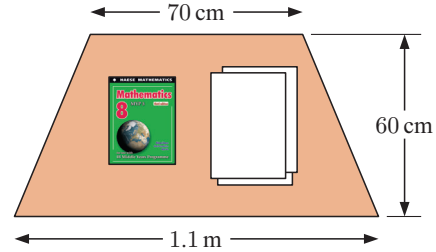
a Find the area of the feature wall.

b Find the area of each:

i red panel **ii** green panel.

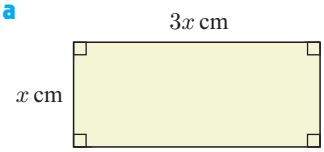
10 **a** Find the area of the top of this desk.

b A classroom has 25 of these desks. The top surface of each desk is to be repainted. Find the total area to be repainted, in m^2.

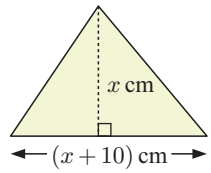


11 Write a formula for the area A of each shape, giving your answers in simplest form:

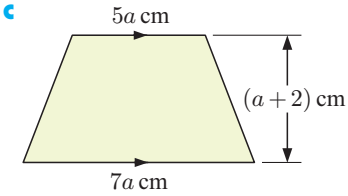
a



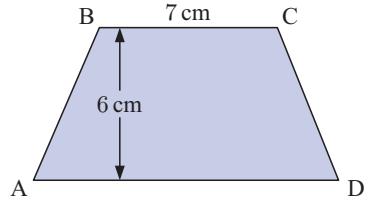
b



c

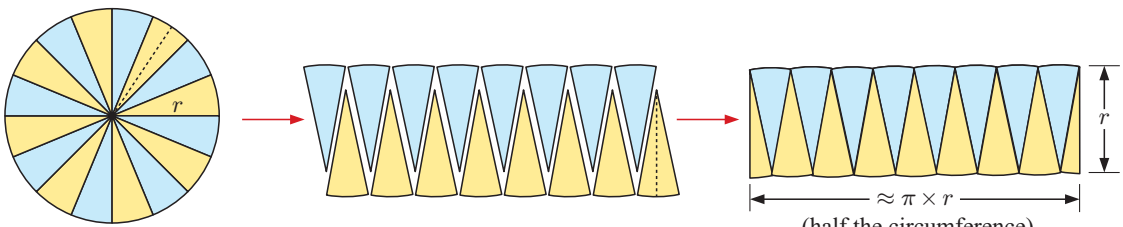


12 The trapezium alongside has area 57 cm^2. Find the length of [AD].



F THE AREA OF A CIRCLE

Consider cutting a circle with radius r into 16 equal sectors and arranging them as shown:



circumference is $2 \times \pi \times r$

The figure obtained closely resembles a rectangle.

- The height is approximately the radius of the circle.
- The top "edge" is approximately the sum of the arc lengths of the blue sectors. This is half the circumference of the circle, which is

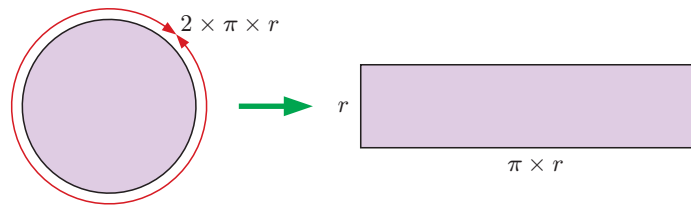
$$\frac{1}{2} \times 2\pi r = \pi \times r.$$

DEMO

PROTRACTOR

Perform this demonstration for yourself. Click on the icon, then print the protractor and cut it into equal sectors.

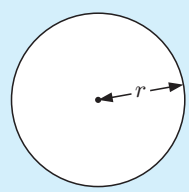
If the original circle is cut into 1000 equal sectors and arranged in the same way, the resulting figure is indistinguishable from a rectangle.

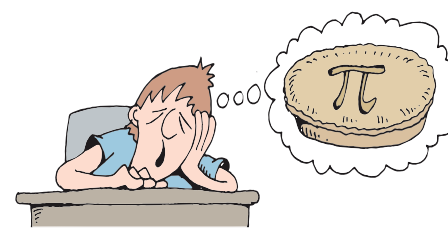


So, the area of the circle = length × width of the rectangle

$$= \pi \times r \times r$$
$$= \pi r^2$$

Area of circle $= \pi r^2$

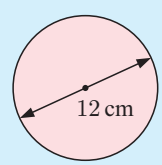




Example 9 **Self Tutor**

Find the area of this circle.

Round your answer to 1 decimal place.



Radius $r = \frac{1}{2} \times$ diameter

$= \frac{1}{2} \times 12$ cm $= 6$ cm

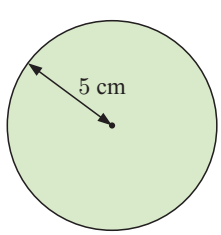
$\therefore$ area $= \pi r^2 = \pi \times 6^2$ cm^2

≈ 113.1 cm^2

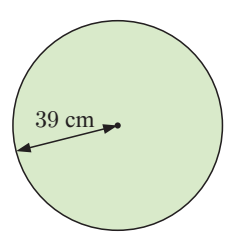
EXERCISE 11F

1 Find the area of each figure. Round your answers to 2 decimal places.

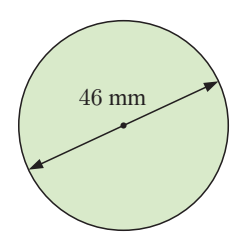
a



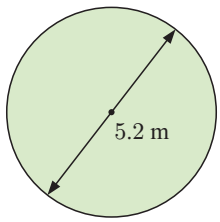
b



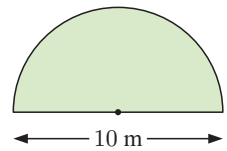
c



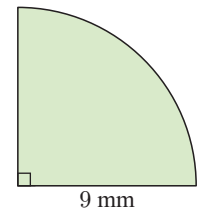
d



e



f

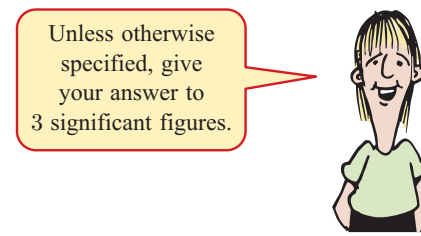


2 A circle has diameter 4.8 cm. Find, to 2 decimal places, its:

a perimeter **b** area.

3 When a water balloon bursts on the ground, it makes a circle of water with diameter 26 cm. Find the area of ground that is wet.

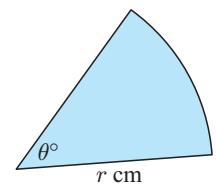
4 A donkey is tethered to a post by a 3.6 m long rope. Over what area can the donkey roam?



5 Look at the sector shown.

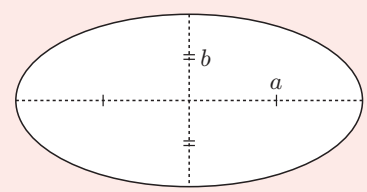
a What fraction of a circle is it?

b Write a formula for the area A of the sector.



ACTIVITY 3 CIRCLES AND ELLIPSES

An ellipse is an oval-shaped figure, obtained by stretching a circle in one direction. The ellipse alongside has **semi-axes** a and b.

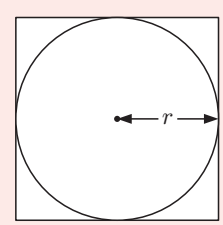


A circle is the special case of an ellipse for which $a = b$.

In this Activity we will see how the properties of circles are related to the properties of ellipses in general.

What to do:

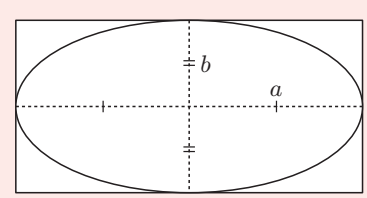
1 In the diagram alongside, a circle with radius r is inscribed in a square.



Write in terms of π, the fraction:

a $\dfrac{\text{area of circle}}{\text{area of square}}$ **b** $\dfrac{\text{perimeter of circle}}{\text{perimeter of square}}$

2 Consider an ellipse with semi-axes a and b inscribed in a rectangle.



a Write a formula for the:

i area of the rectangle

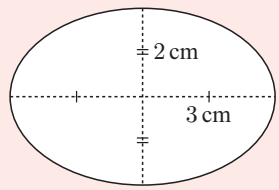
ii perimeter of the rectangle.

b Explain why we might expect the area of the ellipse to be given by $A = \pi ab$.
Research online to confirm that this formula is indeed true for all ellipses.

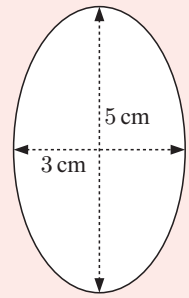
c Explain why we might expect the perimeter of the ellipse to be given by $P = \pi(a+b)$.
Research online to confirm that this formula is *not* true for ellipses in general.
Is it *possible* to write a formula for the perimeter of an ellipse?

3 Use the area formula $A = \pi ab$ to find the area of each ellipse:

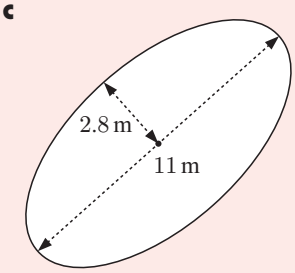
a



b



c



G AREAS OF COMPOSITE FIGURES

Composite figures are figures made up of two or more basic shapes.

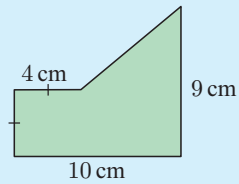
To calculate the area of a composite figure, we divide it into shapes that we are familiar with.

Example 10

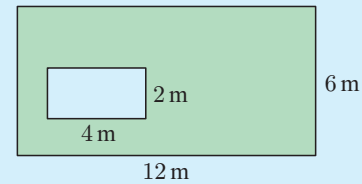
Self Tutor

Find the green shaded area:

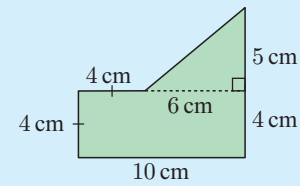
a



b



a We divide the figure into a rectangle and a triangle as shown.



Area = area of rectangle + area of triangle

$= 10 \times 4 \text{ cm}^2 + \frac{1}{2} \times 6 \times 5 \text{ cm}^2$

$= (40 + 15) \text{ cm}^2$

$= 55 \text{ cm}^2$

b Area = area of large rectangle − area of small rectangle

$= 12 \times 6 \text{ m}^2 - 4 \times 2 \text{ m}^2$

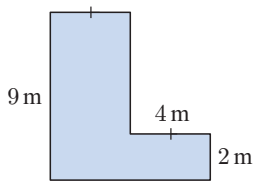
$= (72 - 8) \text{ m}^2$

$= 64 \text{ m}^2$

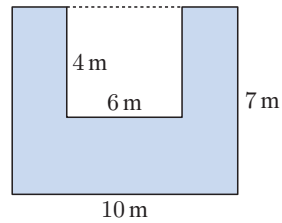
EXERCISE 11G

1 Find the shaded area:

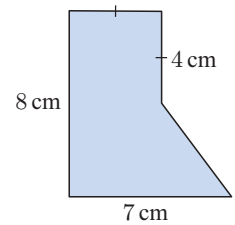
a



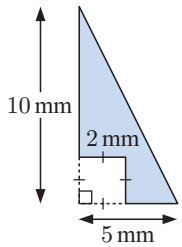
b



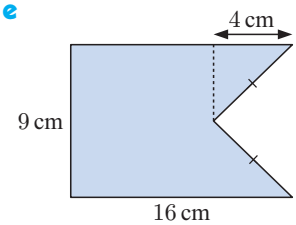
c



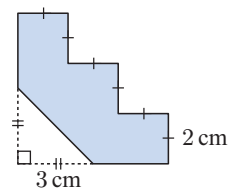
d



e

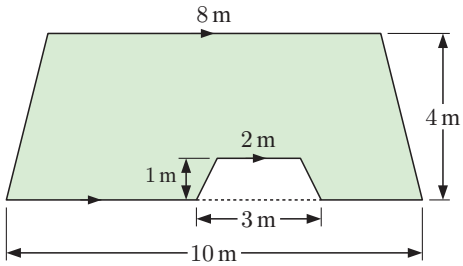


f

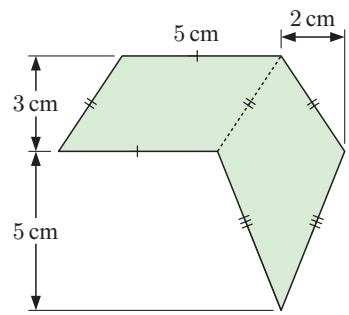


2 Find the shaded area:

a



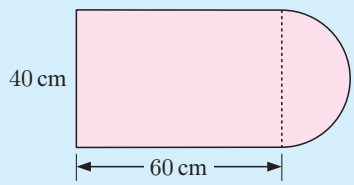
b



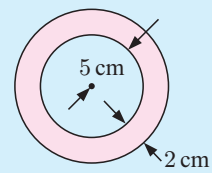
Example 11 Self Tutor

Find, rounded to 1 decimal place, the pink shaded area:

a



b



a Area = area of rectangle + area of semi-circle

$= 40 \times 60 \text{ cm}^2 + \frac{1}{2} \times (\pi \times 20^2) \text{ cm}^2$

$\approx 3028.3 \text{ cm}^2$

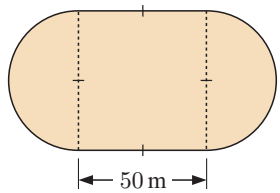
b Area = area of large circle − area of small circle

$= \pi \times 7^2 \text{ cm}^2 - \pi \times 5^2 \text{ cm}^2$

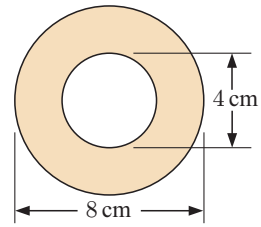
$\approx 75.4 \text{ cm}^2$

3 Find the area of the shaded region, rounding your answer to 1 decimal place:

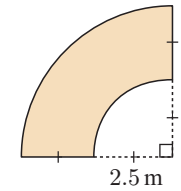
a



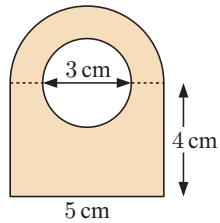
b



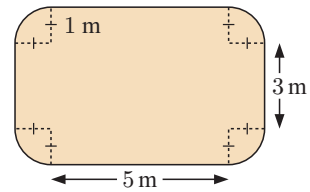
c



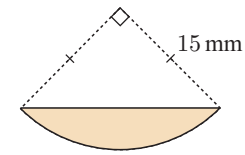
d



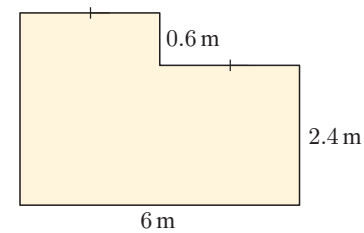
e



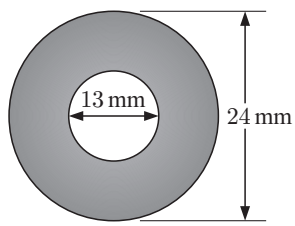
f



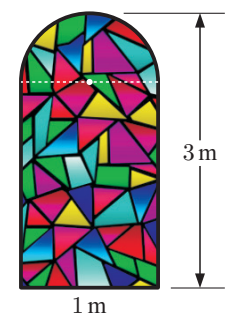
4 The wall shown is to be tiled with 30 cm square tiles. How many tiles are needed?



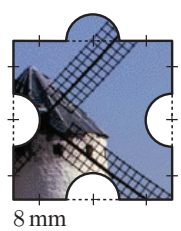
5 A plain metal washer is shown below. Find the area of the top surface.



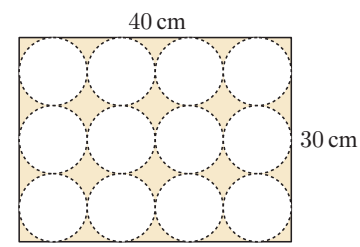
6 Find the area of this stained glass window.



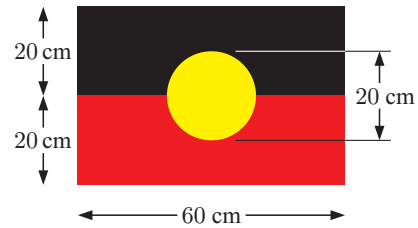
7 Find the area of the jigsaw puzzle piece shown.
Give your answer to the nearest mm^2.



8 Ian cuts 12 circles of pastry from a 30 cm by 40 cm sheet.
Find the area of pastry which remains.



9 An Australian Aboriginal flag is shown alongside.



a Find the area of:

i the flag

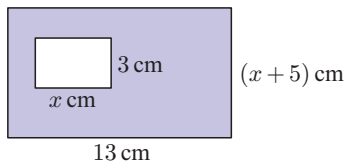
ii the yellow circle.

b What percentage of the flag is yellow?

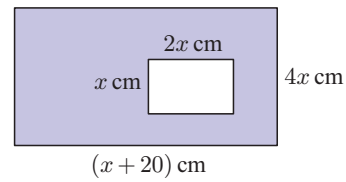
c Find the area of the red part.

10 Write a formula for the area A of each shaded region. Give your answers in simplest form.

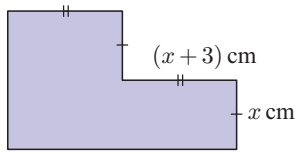
a



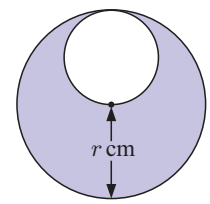
b



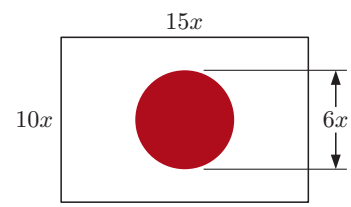
c



d



11 The Japanese flag features a red circular "sundisc" on a white rectangular "field".

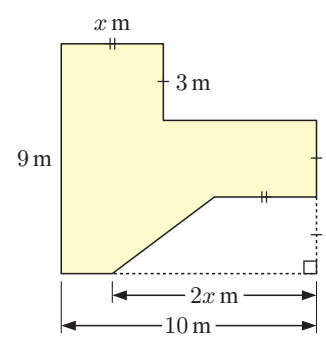


a What fraction of the flag is taken up by the "sundisc"? Write your answer in terms of π.

b What percentage of the flag is the "field"?

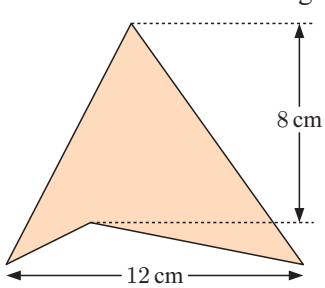
12 **a** Find, in simplest form, the area A of this figure.

b Given that the figure has area 54 m^2, find the value of x.

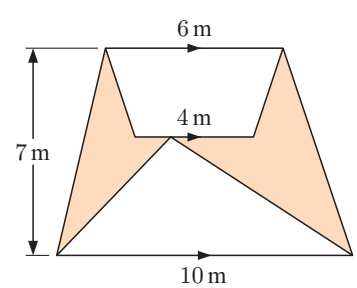


13 Find the area of the shaded region:

a

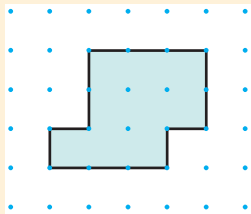


b



INVESTIGATION 2 PICK'S RULE FOR AREAS

This diagram shows a polygon whose **vertices** lie on **lattice points**.



The lattice points which lie *on* the figure are called **boundary points**.

The lattice points which lie *inside* the figure are called **interior points**.

The figure shown has 14 boundary points and 3 interior points.

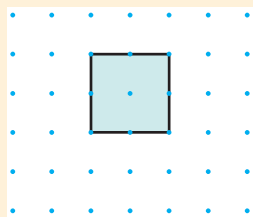
Suppose that for a given polygon:

B = the number of boundary points
I = the number of interior points.

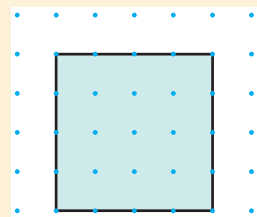
What to do:

1 Copy the table below, then complete it using these figures:

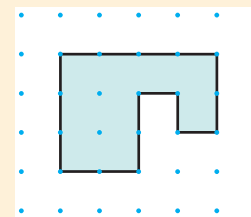
a



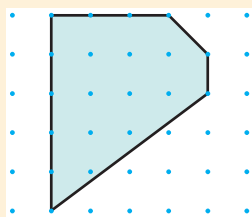
b



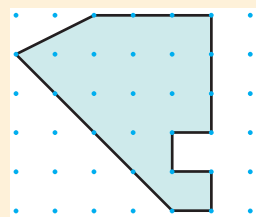
c



d



e



f

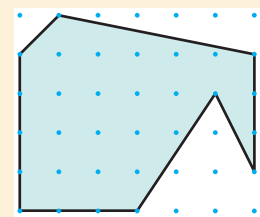


Figure	B	$\frac{B}{2}$	I	$\frac{B}{2}+I$	*Area* A
a					
b					
c					
d					
e					
f					

PRINTABLE TABLE

2 Suggest a rule connecting A, B, and I. This result is called **Pick's Rule**.

3 Draw several polygons of your own on grid paper, and check that your rule is correct.

PRINTABLE DOT GRID PAPER