

- 13** A cyclist rides a bicycle with 88 pedal revolutions per minute. Each pedal revolution results in 3.2 revolutions of the wheels. The wheels have diameter 70 cm. How long will it take the cyclist to travel 40 km?

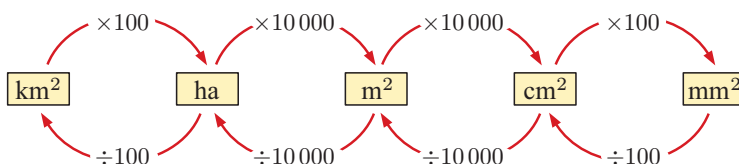
C

UNITS OF AREA

Area is the measurement of the size of a surface.

In the metric system, area is measured using the *squares* of the length units. We also use a unit called a **hectare** (ha).

$$\begin{aligned}
 1 \text{ mm}^2 &= 1 \text{ mm} \times 1 \text{ mm} \\
 1 \text{ cm}^2 &= 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2 \\
 1 \text{ m}^2 &= 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2 \\
 1 \text{ ha} &= 100 \text{ m} \times 100 \text{ m} = 10\,000 \text{ m}^2 \\
 1 \text{ km}^2 &= 1000 \text{ m} \times 1000 \text{ m} = 1\,000\,000 \text{ m}^2 \text{ or } 100 \text{ ha}
 \end{aligned}$$

**Example 5****Self Tutor**

Convert:

a 6 m^2 into cm^2 **b** $18\,500 \text{ m}^2$ into ha.

$$\begin{aligned}
 \text{a} \quad & 6 \text{ m}^2 \\
 &= (6 \times 10\,000) \text{ cm}^2 \\
 &= 60\,000 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad & 18\,500 \text{ m}^2 \\
 &= (18\,500 \div 10\,000) \text{ ha} \\
 &= 1.85 \text{ ha}
 \end{aligned}$$

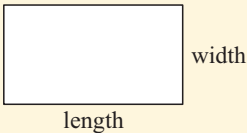
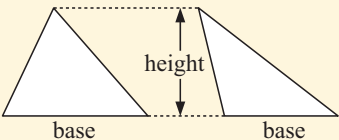
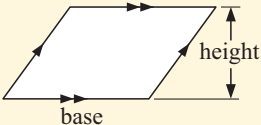
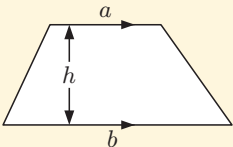
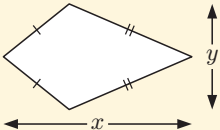
EXERCISE 12C**1** Convert:**a** 23 mm^2 into cm^2 **b** 3.6 ha into m^2 **c** 726 cm^2 into m^2 **d** 7.6 m^2 into mm^2 **e** 8530 m^2 into ha**f** 0.354 ha into cm^2 **g** 13.54 cm^2 into mm^2 **h** 432 m^2 into cm^2 **i** 0.00482 m^2 into mm^2 .**2** Convert:**a** 3 km^2 into m^2 **b** 0.7 km^2 into ha**c** 660 ha into m^2 **d** 660 ha into km^2 **e** 0.05 m^2 into cm^2 **f** 25 cm^2 into m^2 **g** 5.2 mm^2 into cm^2 **h** 0.72 km^2 into mm^2 .

- 3** Sam purchased 2 m^2 of material. She needs to cut it into rectangles of area 200 cm^2 for a patchwork quilt. This can be done with no waste. How many rectangles can Sam cut out?
- 4** I have purchased a 4.2 ha property. Council regulations allow me to have 5 free range chickens for every 100 m^2 . How many free range chickens am I allowed to have?

D

AREA OF POLYGONS

In previous years you should have seen the following area formulae:

Shape	Figure	Formula
Rectangle		$\text{Area} = \text{length} \times \text{width}$
Triangle		$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$
Parallelogram		$\text{Area} = \text{base} \times \text{height}$
Trapezium		$\text{Area} = \left(\frac{a+b}{2} \right) \times h$
Kite		$\text{Area} = \frac{1}{2} \times x \times y$

DEMO



DEMO



DEMO



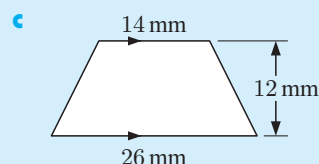
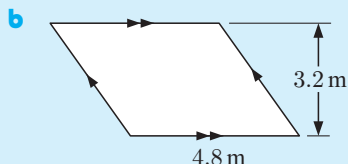
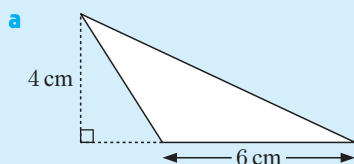
DEMO



Example 6

Self Tutor

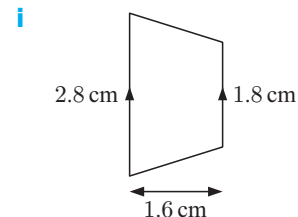
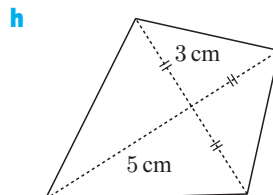
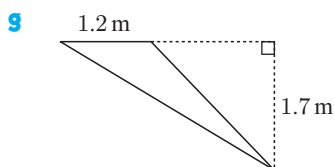
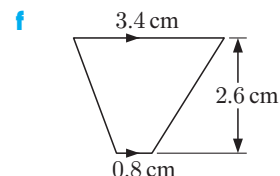
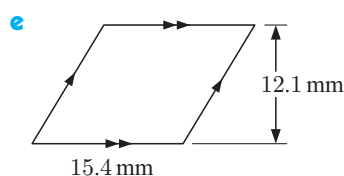
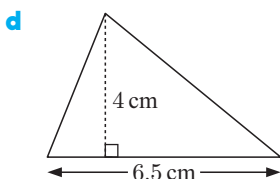
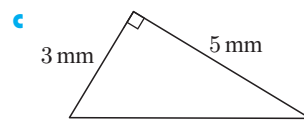
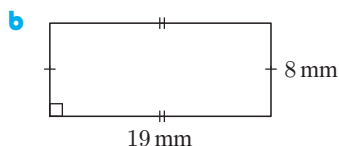
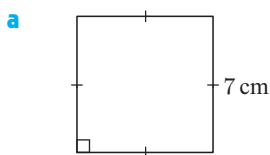
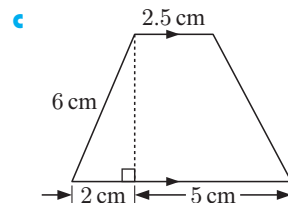
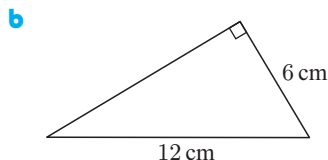
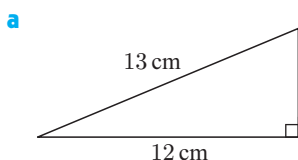
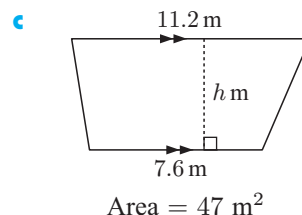
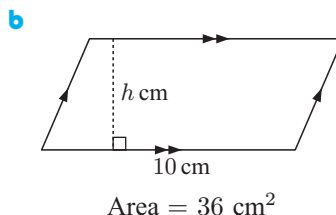
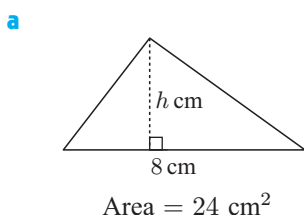
Find the area of each figure:



a Area
 $= \frac{1}{2} \times \text{base} \times \text{height}$
 $= \frac{1}{2} \times 6 \times 4 \text{ cm}^2$
 $= 12 \text{ cm}^2$

b Area
 $= \text{base} \times \text{height}$
 $= 4.8 \times 3.2 \text{ m}^2$
 $= 15.36 \text{ m}^2$

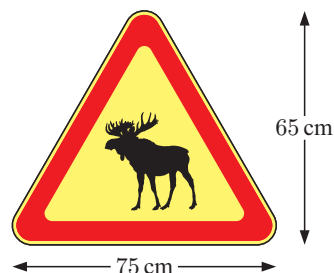
c Area
 $= \left(\frac{a+b}{2} \right) \times h$
 $= \left(\frac{14+26}{2} \right) \times 12 \text{ mm}^2$
 $= 240 \text{ mm}^2$

EXERCISE 12D**1** Find the area of each figure:**2** Find the area of each figure:**3** Calculate the height h of each figure:**4** A council orders 25 road signs to warn motorists to watch out for moose.

a By assuming the signs are triangular, find the area of metal sheeting (in m^2) required for the 25 signs.

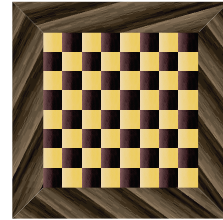
b To allow for wastage when the signs are cut, an extra 20% of the metal is needed. How much metal needs to be purchased?

c The metal costs \$28.40 per m^2 . Find its total cost.

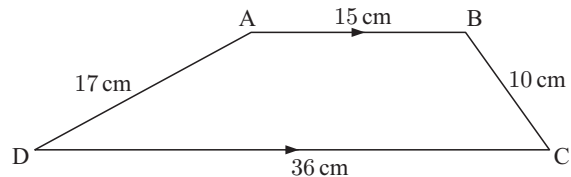


5 A 15 cm by 20 cm rectangle has the same perimeter as a square. Which figure has the greater area? Explain your answer.

- 6 A 4.2 m by 3.5 m tablecloth is used to cover a square table with sides of length 3.1 m. Find the area of the tablecloth which overhangs the edges.
- 7 A square tile has area 256 cm^2 . How many tiles are needed to cover a floor $4 \text{ m} \times 2.4 \text{ m}$?
- 8 Find the area of a rhombus which has diagonals of length 12 cm and 8 cm.
- 9 One diagonal of a rhombus is twice the length of the other. Given that the rhombus has area 32 cm^2 , find the length of the shorter diagonal.
- 10 A chessboard consists of 5 cm squares of blackwood for the black squares, and maple for the white squares. The squares are surrounded by an 8 cm wide blackwood border. Determine the percentage of the board which is made of maple.

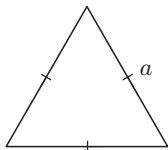


- 11 The area of trapezium ABCD is 204 cm^2 . Find the area of triangle DBC.

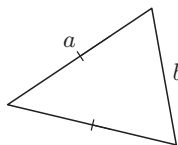


- 12 Find the area of a kite with diagonals of length 16 cm and 10 cm.
- 13 Find the area of an equilateral triangle with side length 4.8 cm.
- 14 Find the side length of an equilateral triangle with area 325 mm^2 .
- 15 Write a formula for the area A of each shape:

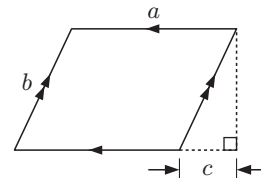
a



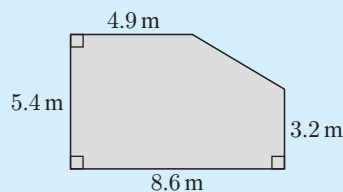
b



c

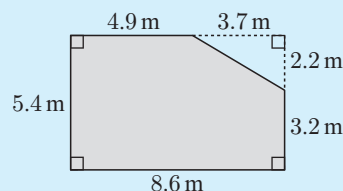
**Example 7****Self Tutor**

Find the shaded area:



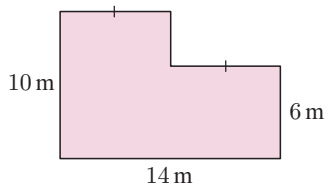
Shaded area

$$\begin{aligned}
 &= \text{area of rectangle} - \text{area of triangle} \\
 &= (8.6 \times 5.4) \text{ m}^2 - \left(\frac{1}{2} \times 3.7 \times 2.2\right) \text{ m}^2 \\
 &= 42.37 \text{ m}^2
 \end{aligned}$$

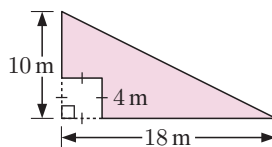


16 Find the shaded area:

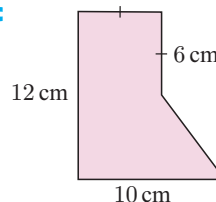
a



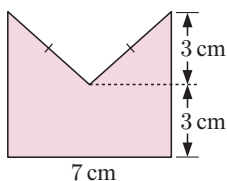
b



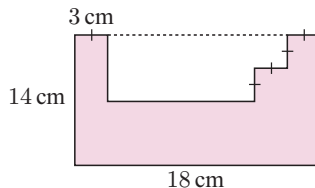
c



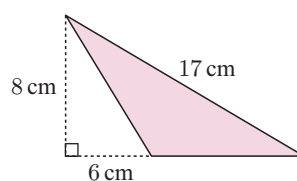
d



e



f



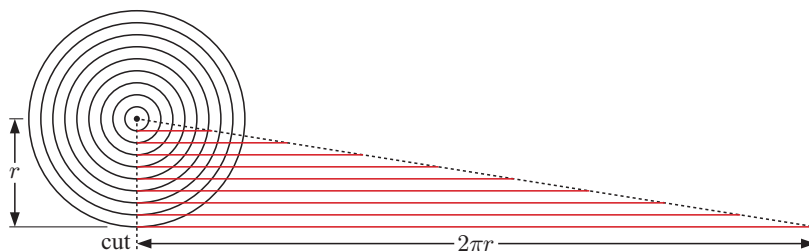
17 A regular hexagon and an equilateral triangle have the same perimeter. Find the ratio of their areas.

E

AREA OF CIRCLES AND SECTORS

Consider drawing concentric circles within a circle with radius r .

We cut along a radius, then “unwrap” the rings to form a triangle.



The height of the triangle is the radius of the circle.

The length of the triangle is the circumference of the circle.

So, the area of the circle = $\frac{1}{2} \times \text{length} \times \text{height of the triangle}$

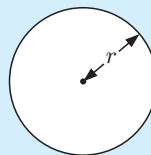
$$= \frac{1}{2} \times 2\pi r \times r$$

$$= \pi r^2$$

DEMO

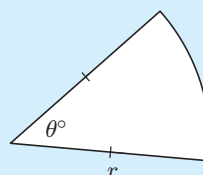


$$\text{Area of circle} = \pi r^2$$



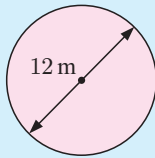
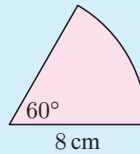
The area of a **sector** is a fraction of the area of the circle it is taken from.

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$



Example 8

Find, to 1 decimal place, the shaded area:

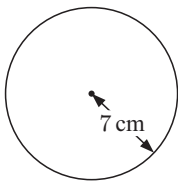
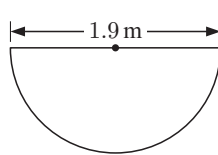
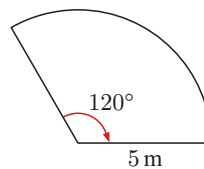
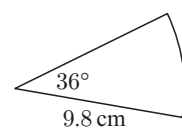
a**b**

$$\begin{aligned}\text{a Area} &= \pi r^2 \\ &= \pi \times 6^2 \\ &= 36\pi \\ &\approx 113.1 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{b Area} &= \frac{\theta}{360} \times \pi r^2 \text{ cm}^2 \\ &= \frac{60}{360} \times \pi \times 8^2 \text{ cm}^2 \\ &\approx 33.5 \text{ cm}^2\end{aligned}$$

EXERCISE 12E

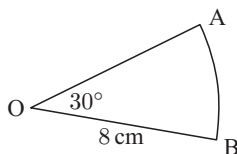
- 1 Find the area of each shape:

a**b****c****d**

- 2 Find, in terms of π , the area of a circle with radius 3 cm.
 3 Calculate, to 2 decimal places, the area of a circle with diameter 9.7 cm.
 4 Calculate the area of a sector with radius 8.7 cm and angle 210° .

- 5 Find, in terms of π , the:

- a** length of arc AB
b perimeter of sector OAB
c area of sector OAB.



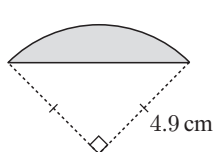
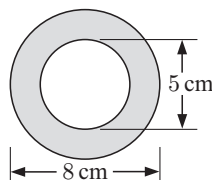
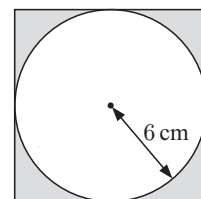
- 6 Calculate the radius of a circle with:

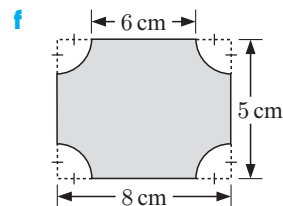
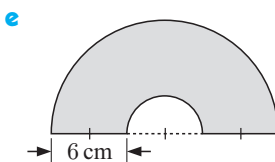
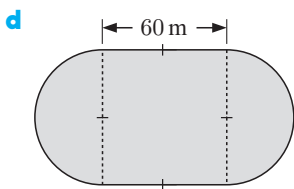
- a** area $9\pi \text{ m}^2$ **b** area 20 cm^2 .

- 7 Find the angle of a sector with area 30 cm^2 and radius 12 cm.

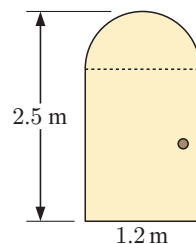
- 8 A circular golfing green has diameter 20 m. The hole must be positioned on the green at least 3 metres from its edge. Find the area of the green on which the hole can be positioned.

- 9 Find the shaded area:

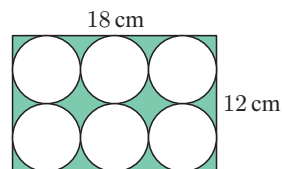
a**b****c**



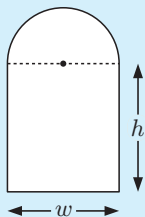
- 10** A door is in the shape of a rectangle surmounted by a semi-circle. The width of the door is 1.2 m, and the height of the door is 2.5 m. Find the total area of the door.



- 11** A restaurant uses square tables with sides of length 1.3 m, and round tablecloths with diameter 2 m. Determine the percentage of each tablecloth which overhangs its table.
- 12** 6 identical metal discs are stamped out of an 18 cm by 12 cm sheet of tin as illustrated. What percentage of the tin is wasted?



- 13** A gravel path 1 m wide is placed around a circular garden bed of diameter 2 m. The gravel costs \$7.90 per m^2 .
- a** Find the area of the path. **b** Find the cost of the gravel.
- 14** Answer the **Opening Problem** on page 170.

Example 9**Self Tutor**

The illustrated door consists of a semi-circle and a rectangle. Find a formula for the area of the door in terms of the width w and height h of the rectangular part.

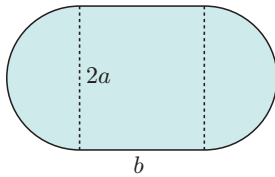
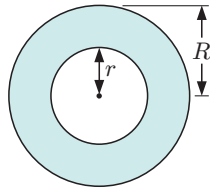
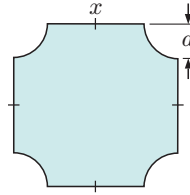
The area of a rectangle = height $\times$ width
 $= hw$

The radius of the semi-circle is $\frac{w}{2}$.

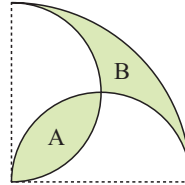
$$\begin{aligned}\therefore \text{ the area of the semi-circle} &= \frac{1}{2} \times \pi r^2 \\ &= \frac{1}{2} \times \pi \times \left(\frac{w}{2}\right)^2 \\ &= \frac{1}{2} \times \pi \times \frac{w^2}{4} \\ &= \frac{1}{8} \pi w^2\end{aligned}$$

$$\therefore \text{ the total area is } A = hw + \frac{1}{8} \pi w^2$$

15 Find a formula for the area A of the following regions:

a**b****c**

16 Show that the two shaded regions have equal area.



QUICK QUIZ



MULTIPLE CHOICE QUIZ

REVIEW SET 12A

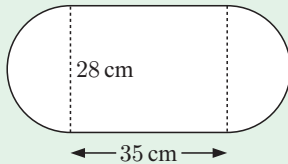
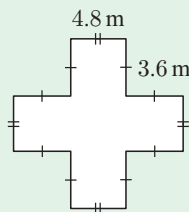
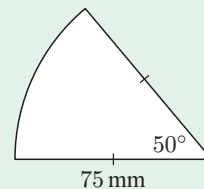
1 Convert:

a 6.7 km into m

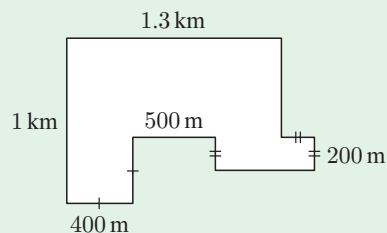
b 9.3 mm into cm

c 2.59 ha into m^2 .

2 Find the perimeter of each figure:

a**b****c**

3 Find the perimeter of this racing track.

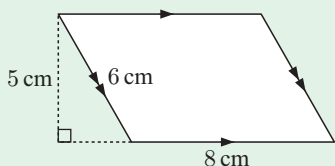


4 A circular playing field has radius 80 metres. The field is surrounded by a fence 10 metres from the edge of the field. Find the length of the fence.

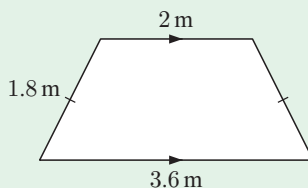
5 In Germany, Frankfurt Airport occupies 23 km^2 of land, whereas Stuttgart Airport occupies 400 ha. How many times larger is Frankfurt Airport than Stuttgart Airport?

6 Find the area of each figure:

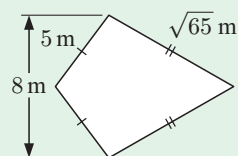
a



b



c



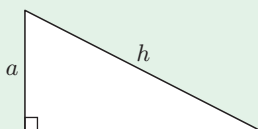
7 Find the area of:

- a** a sector with radius 10 cm and angle 120°
- b** a right angled triangle with base 5 cm and hypotenuse 13 cm
- c** a circle with diameter 15 cm.

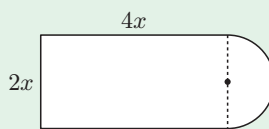
8 A circle has area 8 m^2 . Find its circumference.

9 Write a formula for the area A of each figure:

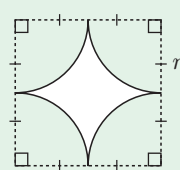
a



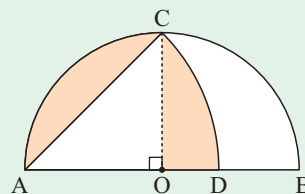
b



c



- 10** In the semi-circle alongside, ACD is a sector of a circle with centre A .
Show that the two shaded regions have equal area.



11 A new food warehouse has floor area 1.4 ha.

- a** Write this area in m^2 .
- b** The concrete floor costs \$90 per square metre. Find its total cost.
- c** The warehouse is rectangular with length 125 m.
 - i** Find the width of the warehouse.
 - ii** Hence find the perimeter of the warehouse.

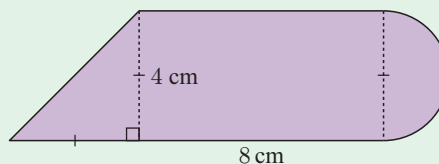
REVIEW SET 12B

1 Convert:

- a** 7260 m into km
- b** 340 cm^2 into mm^2
- c** $38\,200 \text{ mm}^2$ into m^2 .

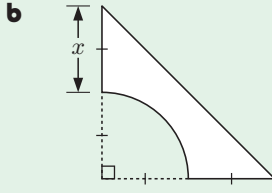
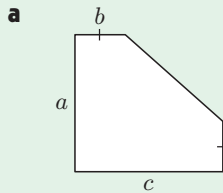
2 12 boxes with height 28 cm are stacked on top of each other.
Find the height of the stack in metres.

3 Find the perimeter of this figure.

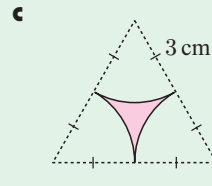
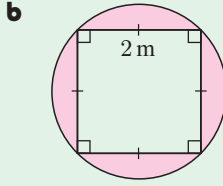
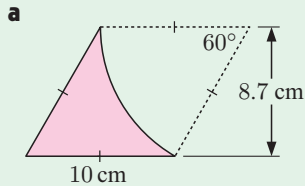


4 Find the angle of a sector with arc length 32 cm and radius 7 cm.

5 Write a formula for the perimeter P of each figure:



6 Find the shaded area:

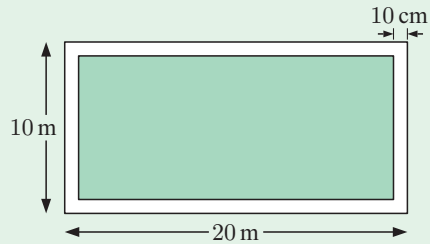


7 Calculate the area in hectares of a rectangular field with sides 1.6 km and 700 m.

8 How many 8 cm by 10 cm tiles are needed to cover a 4.4 m by 6 m floor?

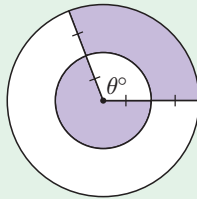
9 The boundary of a sports field has the dimensions shown.

- Find the area of paint used for the boundary.
- The paint costs \$31.50 per m^2 . How much does the boundary cost to paint?



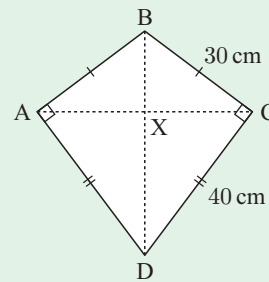
10 Find the side length of an equilateral triangle with area 80 cm^2 .

11 The shaded regions have equal area.
Find the value of θ .



12 A kite manufacturer produces kites with the dimensions shown alongside.

- Find the perimeter of the kite.
- Find the area of the kite.
- Find the length of the long diagonal.
 - Hence find the length of the short diagonal.
- Find the length of:
 - [AX]
 - [BX]
 - [DX]



- Illustrate *two* different ways in which 12 of these kites can be cut from a 1.7 m by 1.2 m piece of material.
 - Find the percentage of the material that is wasted.