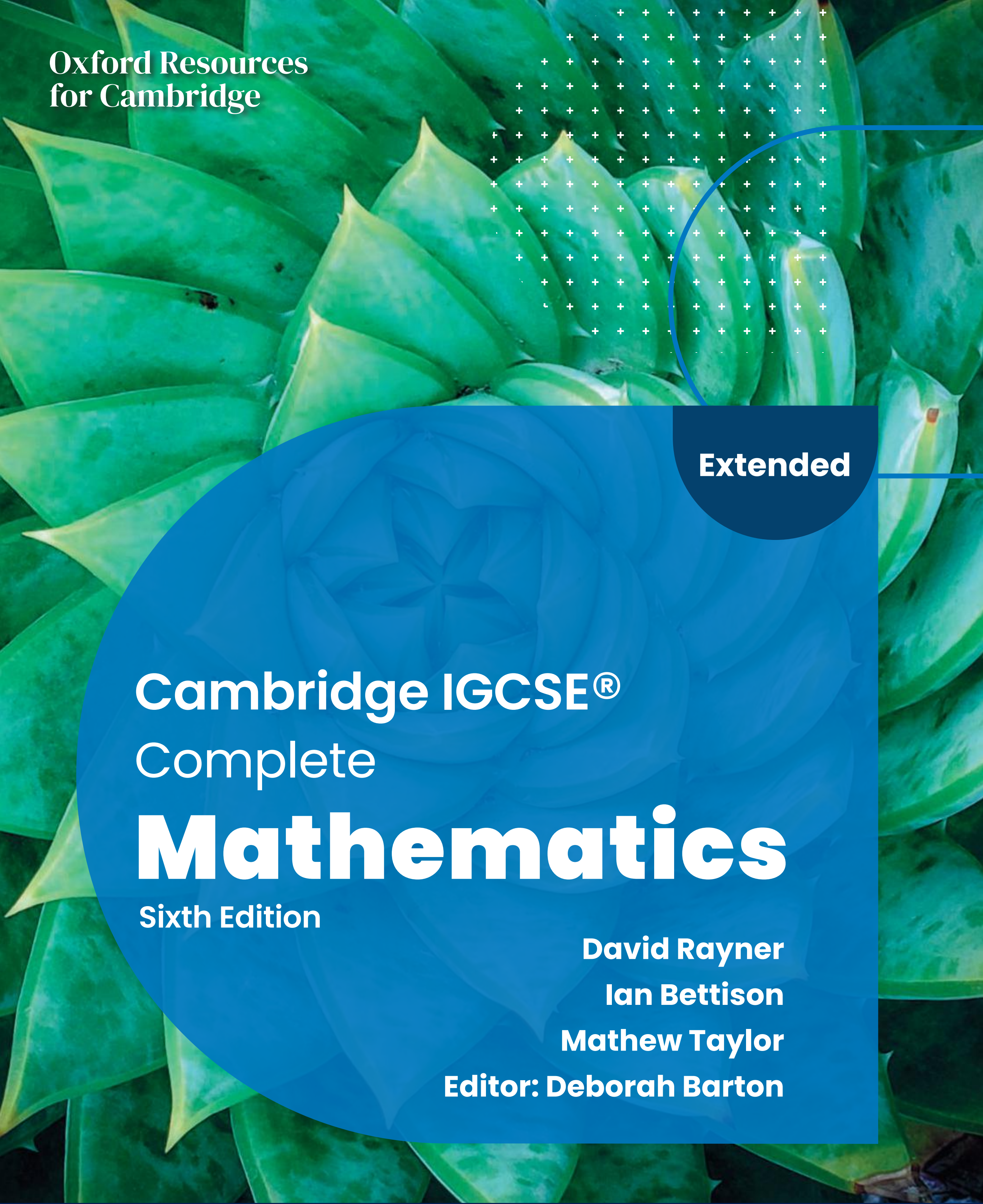

Oxford Resources for Cambridge

Extended

Cambridge IGCSE® Complete Mathematics

Sixth Edition

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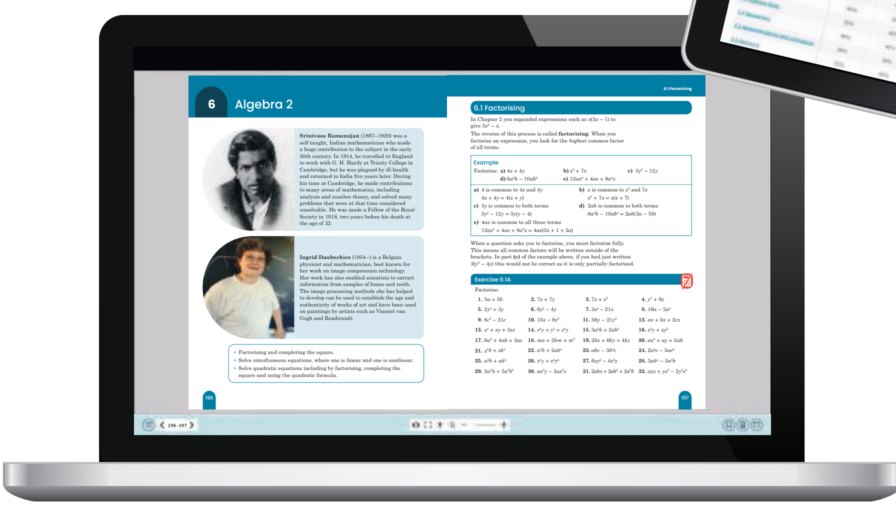

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Contents

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Introduction

About this book

This book is designed specifically for the Cambridge IGCSE® Mathematics course. Experienced examiners have been involved in all aspects of the course, to ensure that the content adheres to the latest syllabus.

Using this book will ensure that you are well prepared for the exam at this level, and also studies beyond the IGCSE level in Mathematics. The features below are designed to make learning as interesting and effective as possible.

Finding your way around

To get the most out of this book when studying or revising, use the:

- **Contents list** to help you find the appropriate units
- **Index** to find key words so you can turn to any concept straight away.

Learning objectives

At the start of each chapter you will find a list of objectives. These will tell you what you should be able to do by the end of the chapter. They are based on what you need to cover for the Cambridge IGCSE syllabus.

Famous mathematicians

These are included at the start of each chapter to give you a brief insight into the life of a mathematician who played an important part in the development of the ideas contained in that chapter.

By finding out about the history of mathematics and considering a topic within the broader context of the subject, you can make connections between topics and develop a greater appreciation of how mathematics has developed over the centuries.

Worked examples

Worked examples are an important feature of the book and can be found in every sub-topic. These show you the important skills techniques required in the exercises below and also provide a model for how to structure your solutions.

Exercises

There are thousands of questions in this book, providing ample opportunities to practise the skills and techniques required in the exam. The exercises contain questions of varying levels of difficulty, so that you can progress through a topic as your knowledge and confidence increases.

Each exercise has an icon to denote whether you can use a calculator or not. This means you can use a calculator, while this means you should not. The same icons also appear in the Revision Exercises.

Revision Exercise

At the end of each chapter, you will find revision questions to bring together all your knowledge and test your understanding of the contents of the chapter.

Examination-style questions

The revision exercises are followed by exam-style practice questions. These are very similar to the kind of questions you should expect to see in the real exam.

Tips

Yellow boxes throughout the exercises provide further information, hints on how to approach a question, or reminders of other concepts.

Answers

These can be found at the back of this book, so you can find out immediately whether or not you have answered a question correctly. Answers to all the numerical problems in the exercises, the review questions, and the exam-style questions are all included.

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Additional support can be found on Kerboodle. There are resources for every sub-topic, including adaptive assessments, personalised Next Steps and data-rich reports. You can also access the Digital Student Book.

2 Algebra 1

Isaac Newton (1642–1727) was an English scientist and mathematician, and a prominent figure in the Scientific Revolution of the 17th century. He went to Trinity College Cambridge in 1661 and by the age of 23 he had made three major discoveries: the nature of colours, calculus and the law of gravitation. He used his version of calculus to give the first satisfactory explanation of the motion of the Sun, the Moon and the stars. Because he was extremely sensitive to criticism, Newton was always very secretive, but he was eventually persuaded to publish his discoveries in 1687.

- Substitute into expressions and formulae.
- Simplify expressions and expand brackets.
- Construct and solving linear equations including those where x appears in the denominator as part of a linear expression.
- Solve simultaneous equations.

2.1 Substitution

In algebra, letters are used to represent numbers. These letters are called *variables*.

Mathematical expressions are made up of one or more terms and operations. A term may be a number, a variable or a combination of both. The expression $5x^2 - 6x + 7$ has three terms:

$5x^2$, $-6x$ and 7

You can evaluate an expression by replacing the variables in the expression with specific values. This is called *substitution*.

For example, when $x = -1$, the expression $5x^2 - 6x + 7$ is evaluated:

$$\begin{aligned} 5(-1)^2 - 6(-1) + 7 &= 5 \times 1 + 6 + 7 \\ &= 18 \end{aligned}$$

Example

When $a = 3$, $b = -2$, and $c = 5$, find the value of:

a) $3a + b$ **b)** $ac + b^2$ **c)** $\dfrac{a + c}{b}$ **d)** $a(c - b)$

a) $$\begin{aligned} 3a + b &= (3 \times 3) + (-2) \\ &= 9 - 2 \\ &= 7 \end{aligned}$$

b) $$\begin{aligned} ac + b^2 &= (3 \times 5) + (-2)^2 \\ &= 15 + 4 \\ &= 19 \end{aligned}$$

c) $$\begin{aligned} \frac{a + c}{b} &= \frac{3 + 5}{-2} \\ &= \frac{8}{-2} \\ &= -4 \end{aligned}$$

d) $$\begin{aligned} a(c - b) &= 3[5 - (-2)] \\ &= 3(7) \\ &= 21 \end{aligned}$$

Note that working *down* the page makes the steps easy to read and easy to follow.

Tip

When substituting, remember to always use BIDMAS.

Exercise 2.1A

Evaluate the following.

For Questions **1** to **12**, $a = 3$, $c = 2$, $e = 5$.

1. $3a - 2$
2. $4c + e$
3. $2c + 3a$
4. $5e - a$
5. $e - 2c$
6. $e - 2a$
7. $4c + 2e$
8. $7a - 5e$
9. $c - e$
10. $10a + c + e$
11. $a + c - e$
12. $a - c - e$

For Questions **13** to **24**, $h = 3$, $m = -2$, $t = -3$.

13. $2m - 3$
14. $4t + 10$
15. $3h - 12$
16. $6m + 4$
17. $9t - 3$
18. $4h + 4$
19. $2m - 6$
20. $m + 2$
21. $3h + m$
22. $t - h$
23. $4m + 2h$
24. $3t - m$

For Questions **25** to **36**, $x = -2$, $y = -1$, $k = 0$.

25. $3x + 1$
26. $2y + 5$
27. $6k + 4$
28. $3x + 2y$
29. $2k + x$
30. xy
31. xk
32. $2xy$
33. $2(x + k)$
34. $3(k + y)$
35. $5x - y$
36. $3k - 2x$

Tip

$2x^2$ means $2(x^2)$

$(2x)^2$ means 'work out $2x$ and *then* square it'

$-7x$ means $-7(x)$

$-x^2$ means $-(x^2)$

Example

When $x = -2$, find the value of:

a) $2x^2 - 5x$ **b)** $(3x)^2 - x^2$

a) $2x^2 - 5x = 2(-2)^2 - 5(-2)$
$= 2(4) + 10$
$= 18$

b) $(3x)^2 - x^2 = (3 \times -2)^2 - 1(-2)^2$
$= (-6)^2 - 1(4)$
$= 36 - 4$
$= 32$

Exercise 2.1B

If $x = -3$ and $y = 2$, evaluate:

1. x^2 **2.** $3x^2$ **3.** y^2 **4.** $4y^2$

5. $(2x)^2$ **6.** $2x^2$ **7.** $10 - x^2$ **8.** $10 - y^2$

9. $20 - 2x^2$ **10.** $20 - 3y^2$ **11.** $5 + 4x$ **12.** $x^2 - 2x$

13. $y^2 - 3x^2$ **14.** $x^2 - 3y$ **15.** $(2x)^2 - y^2$ **16.** $4x^2$

17. $(4x)^2$ **18.** $1 - x^2$ **19.** $y - x^2$ **20.** $x^2 + y^2$

21. $x^2 - y^2$ **22.** $2 - 2x^2$ **23.** $(3x)^2 + 3$ **24.** $11 - xy$

25. $12 + xy$ **26.** $(2x)^2 - (3y)^2$ **27.** $2 - 3x^2$ **28.** $y^2 - x^2$

29. $x^2 + y^3$ **30.** $\frac{x}{y}$ **31.** $10 - 3x$ **32.** $2y^2$

33. $25 - 3y$ **34.** $(2y)^2$ **35.** $-7 + 3x$ **36.** $-8 + 10y$

37. $(xy)^2$ **38.** xy^2 **39.** $-7 + x^2$ **40.** $17 + xy$

41. $-5 - 2x^2$ **42.** $10 - (2x)^2$ **43.** $x^2 + 3x + 5$ **44.** $2x^2 - 4x + 1$

45. $\frac{x^2}{y}$

Example

When $a = -2$, $b = 3$, $c = -3$, evaluate:

a) $\frac{2a\,(b^2 - a)}{c}$ **b)** $\sqrt{(a^2 + b^2)}$

a) $(b^2 - a) = 9 - (-2)$
$= 11$

$\therefore \frac{2a\,(b^2 - a)}{c} = \frac{2 \times (-2) \times (11)}{-3}$

$= \frac{-44}{-3}$

$= \frac{44}{3}$

$= 14\frac{2}{3}$

b) $\sqrt{(a^2 + b^2)} = \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

Tip

In mathematics, the $\therefore$ symbol means ‘therefore’.

Exercise 2.1C

Evaluate the following expressions.

For Questions **1** to **16**, $a = 4,\ b = -2,\ c = -3.$

1. $a(b + c)$

2. $a^2\ (b - c)$

3. $2c(a - c)$

4. $b^2\ (2a + 3c)$

5. $c^2(b - 2a)$

6. $2a^2(b + c)$

7. $2(a + b + c)$

8. $3c(a - b - c)$

9. $b^2 + 2b + a$

10. $c^2 - 3c + a$

11. $2b^2 - 3b$

12. $\sqrt{\left(a^2 + c^2\right)}$

13. $\sqrt{\left(ab + c^2\right)}$

14. $\sqrt{\left(c^2 - b^2\right)}$

15. $\dfrac{b^2}{a} + \dfrac{2c}{b}$

16. $\dfrac{c^2}{b} + \dfrac{4b}{a}$

For Questions **17** to **32**, $k = -3,\ m = 1,\ n = -4.$

17. $k^2(2m - n)$

18. $5m\sqrt{\left(k^2 + n^2\right)}$

19. $\sqrt{(kn + 4m)}$

20. $kmn(k^2 + m^2 + n^2)$

21. $k^2m^2(m - n)$

22. $k^2 - 3k + 4$

23. $m^3 + m^2 + n^2 + n$

24. $k^3 + 3k$

25. $m(k^2 - n^2)$

26. $m\sqrt{(k - n)}$

27. $100k^2 + m$

28. $m^2(2k^2 - 3n^2)$

29. $\dfrac{2k + m}{k - n}$

30. $\dfrac{kn - k}{2m}$

31. $\dfrac{3k + 2m}{2n - 3k}$

32. $\dfrac{k + m + n}{k^2 + m^2 + n^2}$

For Questions **33** to **48**, $w = -2,\ x = 3,\ y = 0,\ z = -\dfrac{1}{2}$

33. $\dfrac{w}{z} + x$

34. $\dfrac{w + x}{z}$

35. $y\left(\dfrac{x + z}{w}\right)$

36. $x^2\ (z + wy)$

37. $x\sqrt{\left(x + wz\right)}$

38. $w^2\sqrt{\left(z^2 + y^2\right)}$

39. $2(w^2 + x^2 + y^2)$

40. $2x(w - z)$

41. $\dfrac{z}{w} + x$

42. $\dfrac{z + w}{x}$

43. $\dfrac{x + w}{z^2}$

44. $\dfrac{y^2 - w^2}{xz}$

45. $z^2 + 4z + 5$

46. $\dfrac{1}{w} + \dfrac{1}{z} + \dfrac{1}{x}$

47. $\dfrac{4}{z} + \dfrac{10}{w}$

48. $\dfrac{yz - xw}{xz - w}$

49. Find $K = \sqrt{\left(\dfrac{a^2 + b^2 + c^2 - 2c}{a^2 + b^2 + 4c}\right)}$ when $a = 3,\ b = -2,\ c = -1.$

50. Find $W = \dfrac{kmn(k + m + n)}{(k + m)(k + n)}$ when $k = \dfrac{1}{2},\ m = -\dfrac{1}{3},\ n = \dfrac{1}{4}$

When a calculation is repeated many times, it is often helpful to use a formula. An example of a scientific formula is the formula for converting between degrees Celsius and degrees Fahrenheit. An example of a mathematical formula is the one for calculating the volume of a sphere.

Example 1

Use the formula $F = \frac{9}{5}C + 3$ to convert 45 °C to degrees Fahrenheit.

If $C = 45$, then $F = \frac{9}{5} \times 45 + 32$ 113 °F.

Example 2

Use the formula $V = \frac{4}{3}\pi r^3$ to calculate the volume of a sphere with diameter 12 cm.

Leave your answer in terms of π.

The diameter is 12 cm, so the radius is 6 cm.

So $V = \frac{4}{3}\pi \times 6^3 = 288\pi \text{ cm}^3$

Tip

Rearranging the formula to convert degrees Fahrenheit to degrees Celsius will be covered in Chapter 8: Changing the subject of a formula.

Exercise 2.1D

1. The final speed v of a car is given by the formula $v = u + at$.

   [u = initial speed, a = acceleration, t = time taken]

   Find v when $u = 15$ m/s, $a = 0.2$ m/s^2, $t = 30$ s.

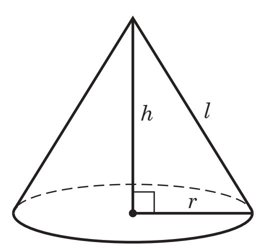
2. The period T of a simple pendulum is given by the formula $T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$, where l is the length of the pendulum and g is the gravitational acceleration. Find T when $l = 0.65$ m, $g = 9.81$ m/s^2 and $\pi = 3.142$.

3. The total surface area A of a cone is related to the radius r and the slant height l by the formula $A = \pi r(r + l)$. Find A when $r = 7$ cm and $l = 11$ cm.

4. The sum S of the squares of the integers from 1 to n is given by $S = \frac{1}{6}n\,(n + 1)\,(2n + 1)$. Find S when $n = 12$.

Tip

The period of a pendulum is the time it takes to complete one full cycle: a left swing and a right swing.

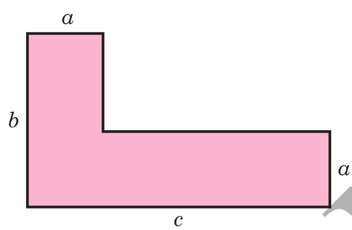


5. The acceleration a of a train is found using the formula
$a = \dfrac{v^2 - u^2}{2s}$.
Find a when $v = 20$ m/s, $u = 9$ m/s and $s = 2.5$ m.

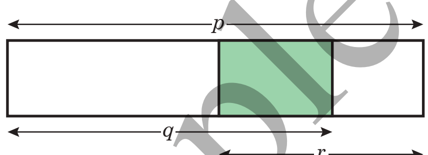
6. Einstein's famous equation relating energy, mass and the speed of light is $E = mc^2$.
Find E when $m = 0.0001$ kg and $c = 3 \times 10^8$ m/s.

7. The distance s travelled by an accelerating rocket is given by $s = ut + \frac{1}{2}at^2$.
Find s when $u = 3$ m/s, $t = 100$ s and $a = 0.1$ m/s^2.

8. Find a formula for the area of the shape below, in terms of a, b and c.



9. Find a formula for the length of the shaded part below, in terms of p, q and r.



Tip

You can find out more about area in Chapter 5.

2.2 Brackets and simplifying

A term outside a pair of brackets multiplies each of the terms inside the brackets. This is the *distributive law*.

Example 1

$3(x - 2y) = 3x - 6y$

Example 2

$2x(x - 2y + z) = 2x^2 - 4xy + 2xz$

Example 3

$7y - 4(2x - 3) = 7y - 8x + 12$

In general, like terms can be added:

x terms can be added to x terms

y terms can be added to y terms

x^2 terms can be added to x^2 terms

But they must not be mixed.

Example 4

$$\begin{aligned} 2x + 3y + 3x^2 + 2y - x &= 2x - x + 3y + 2y + 3x^2 \\ &= x + 5y + 3x^2 \end{aligned}$$

You can rearrange the expression to group together like terms.

Example 5

$$\begin{aligned} 7x + 3x(2x - 3) &= 7x + 6x^2 - 9x \\ &= 6x^2 - 2x \end{aligned}$$

Exercise 2.2A

Simplify these expressions as far as possible.

1. $3x + 4y + 7y$

2. $4a + 7b - 2a + b$

3. $3x - 2y + 4y$

4. $2x + 3x + 5$

5. $7 - 3x + 2 + 4x$

6. $5 - 3y - 6y - 2$

7. $5x + 2y - 4y - x^2$

8. $x^2 - 2 + 3x + x^2 + 7$

9. $2x - 7y - 2x - 3y$

10. $4a + 3a^2 - 2a$

11. $1 + 7a - 8a^2 + 6 + a^2$

12. $x^2 + 3x^2 - 4x^2 + 5x$

13. $\frac{3}{a} + b + \frac{7}{a} - 2b$

14. $\frac{4}{x} - \frac{7}{y} + \frac{1}{x} + \frac{2}{y}$

15. $\frac{m}{x} + \frac{2m}{x}$

16. $\frac{5}{x} - \frac{7}{x} + \frac{1}{2}$

17. $\frac{3}{a} + b + \frac{2}{a} + 2b$

18. $\frac{n}{4} - \frac{m}{3} - \frac{n}{2} + \frac{m}{3}$

19. $x^3 + 7x^2 - 2x^3$

20. $(2x)^2 - 2x^2$

21. $(3y)^2 + x^2 - (2y)^2$

22. $(2x)^2 - (2y)^2 - (4x)^2$

23. $5x - 7x^2 - (2x)^2$

24. $\frac{3}{x^2} + \frac{5}{x^2}$

Expand the brackets and collect like terms to simplify each expression.

25. $3x + 2(x + 1)$

26. $5x + 7(x - 1)$

27. $7 + 3(x - 1)$

28. $9 - 2(3x - 1)$

29. $3x - 4(2x + 5)$

30. $5x - 2x(x - 1)$

31. $7x + 3x(x - 4)$

32. $4(x - 1) - 3x$

33. $5x(x + 2) + 4x$

34. $3x(x - 1) - 7x^2$

35. $3a + 2(a + 4)$

36. $4a - 3(a - 3)$

37. $3ab - 2a(b - 2)$

38. $3y - y(2 - y)$

39. $3x - (x + 2)$

40. $7x - (x - 3)$

41. $5x - 2(2x + 2)$

42. $3(x - y) + 4(x + 2y)$

43. $x(x - 2) + 3x(x - 3)$

44. $3x(x + 4) - x(x - 2)$

45. $y(3y - 1) - (3y - 1)$

46. $7(2x + 2) - (2x + 2)$

47. $7b(a + 2) - a(3b + 3)$

48. $3(x - 2) - (x - 2)$

Two pairs of brackets

To expand two pairs of brackets, multiply each term in the first pair of brackets by each term in the second pair.

Example 1

Expand $(x + 5)(x + 3)$

$$\begin{aligned}(x + 5)(x + 3) &= x(x + 3) + 5(x + 3) \\ &= x^2 + 3x + 5x + 15 \\ &= x^2 + 8x + 15\end{aligned}$$

(Multiply each term in the second bracket by x and by 5.)

Example 2

$$\begin{aligned}(2x - 3)(4y + 3) &= 2x(4y + 3) - 3(4y + 3) \\ &= 8xy + 6x - 12y - 9\end{aligned}$$

Example 3

$$\begin{aligned}3(x + 1)(x - 2) &= 3[x(x - 2) + 1(x - 2)] \\ &= 3[x^2 - 2x + x - 2] \\ &= 3x^2 - 3x - 6\end{aligned}$$

Exercise 2.2B

Expand the brackets and simplify:

1. $(x+1)(x+3)$	**2.** $(x+3)(x+2)$	**3.** $(y+4)(y+5)$
4. $(x-3)(x+4)$	**5.** $(x+5)(x-2)$	**6.** $(x-3)(x-2)$
7. $(a-7)(a+5)$	**8.** $(z+9)(z-2)$	**9.** $(x-3)(x+3)$
10. $(k-11)(k+11)$	**11.** $(2x+1)(x-3)$	**12.** $(3x+4)(x-2)$
13. $(2y-3)(y+1)$	**14.** $(7y-1)(7y+1)$	**15.** $(3x-2)(3x+2)$
16. $(3a+b)(2a+b)$	**17.** $(3x+y)(x+2y)$	**18.** $(2b+c)(3b-c)$
19. $(5x-y)(3y-x)$	**20.** $(3b-a)(2a+5b)$	**21.** $2(x-1)(x+2)$
22. $3(x-1)(2x+3)$	**23.** $4(2y-1)(3y+2)$	**24.** $2(3x+1)(x-2)$
25. $4(a+2b)(a-2b)$	**26.** $x(x-1)(x-2)$	**27.** $2x(2x-1)(2x+1)$
28. $3y(y-2)(y+3)$	**29.** $x(x+y)(x+z)$	**30.** $3z(a+2m)(a-m)$

Be careful with an expression like $(x-3)^2$.
It is not $x^2 - 9$ or even $x^2 + 9$.

$$
\begin{aligned}
(x-3)^2 &= (x-3)(x-3) \\
&= x(x-3) - 3(x-3) \\
&= x^2 - 6x + 9
\end{aligned}
$$

Another common mistake occurs with an expression like $4 - (x-1)^2$.

A common error is to forget that to multiply a set of brackets by -1, you need to change the sign of *all* terms inside the brackets. The following work is correct.

$$
\begin{aligned}
4 - (x-1)^2 &= 4 - 1(x-1)(x-1) \\
&= 4 - 1(x^2 - 2x + 1) \\
&= 4 - x^2 + 2x - 1 \\
&= 3 + 2x - x^2
\end{aligned}
$$

Using a bracket here helps to get the signs correct.

Exercise 2.2C

Expand the brackets and simplify:

1. $(x+4)^2$	**2.** $(x+2)^2$	**3.** $(x-2)^2$
4. $(2x+1)^2$	**5.** $(y-5)^2$	**6.** $(3y+1)^2$

7. $(x + y)^2$

8. $(2x + y)^2$

9. $(a - b)^2$

10. $(2a - 3b)^2$

11. $3(x + 2)^2$

12. $(3 - x)^2$

13. $(3x + 2)^2$

14. $(a - 2b)^2$

15. $(x + 1)^2 + (x + 2)^2$

16. $(x - 2)^2 + (x + 3)^2$

17. $(x + 2)^2 + (2x + 1)^2$

18. $(y - 3)^2 + (y - 4)^2$

19. $(x + 2)^2 - (x - 3)^2$

20. $(x - 3)^2 - (x + 1)^2$

21. $(y - 3)^2 - (y + 2)^2$

22. $(2x + 1)^2 - (x + 3)^2$

23. $3(x + 2)^2 - (x + 4)^2$

24. $2(x - 3)^2 - 3(x + 1)^2$

Three pairs of brackets

To expand three pairs of brackets, expand the first two pairs of brackets, and then multiply this result by the third pair.

Example

$$\begin{aligned}(x + 1)(x + 2)(x + 3) &= [x(x + 2) + 1(x + 2)](x + 3)\\ &= [x^2 + 2x + x + 2](x + 3)\\ &= (x^2 + 3x + 2)(x + 3)\\ &= x(x^2 + 3x + 2) + 3(x^2 + 3x + 2)\\ &= x^3 + 3x^2 + 2x + 3x^2 + 9x + 6\\ &= x^3 + 6x^2 + 11x + 6\end{aligned}$$

Exercise 2.2D

Expand the brackets and simplify.

1. $(x + 2)(x - 3)(x - 4)$

2. $(x - 1)(x + 2)(x - 5)$

3. $(x + 6)(x - 3)(x + 5)$

4. $(2x - 1)(x + 1)(x - 1)$

5. $(3x + 1)(2x + 1)(x - 2)$

6. $(x + 2)(4x - 3)(2x + 3)$

7. $(6x - 5)(2x + 7)(3x - 8)$

8. $(x + 1)^2(x - 4)$

9. $(x - 3)(x - 2)^2$

10. $(x - 1)(2x + 3)^2$

11. $(x - 1)^3$

12. $(3x + 2)^3$

13. $(x - 2)^3 - (x + 1)^3$

14. $(x + 3)^3 - (x - 4)^3$

15. $(2x + 1)^3 + 3(x + 1)^3$

2.3 Solving linear equations

If an equation contains only one variable, and the highest power of that variable is 1, then the equation is a *linear equation*. In this section you are going to solve linear equations.

Here are some examples, illustrating a few of the techniques you may use.

- If the x term is negative, add an x term with a positive coefficient to both sides of the equation.

Example 1

Solve $4 - 3x = 2$

$4 = 2 + 3x$ (Add $3x$ to both sides.)

$2 = 3x$ (Subtract 2 from both sides.)

$\frac{2}{3} = x$ (Divide both sides by 3.)

- If there are x terms on both sides, collect them on one side and then simplify.

Example 2

Solve $2x - 7 = 5 - 3x$

$2x + 3x = 5 + 7$ (Add $3x$ to both sides.)

$5x = 12$

$x = \frac{12}{5} = 2\frac{2}{5}$ (Divide both sides by 5 and simplify.)

- If there is a fraction in the x term, multiply out to simplify the equation.

Example 3

Solve $\frac{2x}{3} = 10$

$2x = 30$ (Multiply both sides by 3.)

$x = \frac{30}{2} = 15$ (Divide both sides by 2 and simplify.)

Exercise 2.3A

Solve:

1. $2x - 5 = 11$ **2.** $3x - 7 = 20$ **3.** $2x + 6 = 20$ **4.** $5x + 10 = 60$

5. $8 = 7 + 3x$ **6.** $12 = 2x - 8$ **7.** $-7 = 2x - 10$ **8.** $3x - 7 = -10$

9. $12 = 15 + 2x$

10. $5 + 6x = 7$

11. $\frac{x}{5} = 7$

12. $\frac{x}{10} = 13$

13. $7 = \frac{x}{2}$

14. $\frac{x}{2} = \frac{1}{3}$

15. $\frac{3x}{2} = 5$

16. $\frac{4x}{5} = -2$

17. $7 = \frac{7x}{3}$

18. $\frac{3}{4} = \frac{2x}{3}$

19. $\frac{5x}{6} = \frac{1}{4}$

20. $-\frac{3}{4} = \frac{3x}{5}$

21. $\frac{x}{2} + 7 = 12$

22. $\frac{x}{3} - 7 = 2$

23. $\frac{x}{5} - 6 = -2$

24. $4 = \frac{x}{2} - 5$

25. $10 = 3 + \frac{x}{4}$

26. $\frac{a}{5} - 1 = -4$

27. $100x - 1 = 98$

28. $7 = 7 + 7x$

29. $\frac{x}{100} + 10 = 20$

30. $1000x - 5 = -6$

31. $-4 = -7 + 3x$

32. $2x + 4 = x - 3$

33. $x - 3 = 3x + 7$

34. $5x - 4 = 3 - x$

35. $4 - 3x = 1$

36. $5 - 4x = -3$

37. $7 = 2 - x$

38. $3 - 2x = x + 12$

39. $6 + 2a = 3$

40. $a - 3 = 3a - 7$

41. $2y - 1 = 4 - 3y$

42. $7 - 2x = 2x - 7$

43. $7 - 3x = 5 - 2x$

44. $8 - 2y = 5 - 5y$

45. $x - 16 = 16 - 2x$

46. $x + 2 = 3.1$

47. $-x - 4 = -3$

48. $-3 - x = -5$

49. $-\frac{x}{2} + 1 = -\frac{1}{4}$

50. $-\frac{3}{5} + \frac{x}{10} = -\frac{1}{5} - \frac{x}{5}$

Equations with brackets

Example

Solve $x - 2(x - 1) = 1 - 4(x + 1)$ (Expand the brackets.)

$$x - 2x + 2 = 1 - 4x - 4$$ (Be careful to get the sign of each term correct.)

$$x - 2x + 4x = 1 - 4 - 2$$ (Add $4x$ to both sides.)

$$3x = -5$$ (Simplify.)

$$x = -\frac{5}{3}$$ (Divide both sides by 3.)

Exercise 2.3B

Solve:

1. $x + 3(x + 1) = 2x$

2. $1 + 3(x - 1) = 4$

3. $2x - 2(x + 1) = 5x$

4. $2(3x - 1) = 3(x - 1)$

5. $4(x - 1) = 2(3 - x)$

6. $4(x - 1) - 2 = 3x$

7. $4(1 - 2x) = 3(2 - x)$

8. $3 - 2(2x + 1) = x + 17$

9. $4x = x - (x - 2)$

10. $7x = 3x - (x + 20)$

11. $5x - 3(x - 1) = 39$

12. $3x + 2(x - 5) = 15$

13. $7 - (x + 1) = 9 - (2x - 1)$

14. $10x - (2x + 3) = 21$

15. $3(2x + 1) + 2(x - 1) = 23$

16. $5(1 - 2x) - 3(4 + 4x) = 0$

17. $7x - (2 - x) = 0$

18. $3(x + 1) = 4 - (x - 3)$

19. $3y + 7 + 3(y - 1) = 2(2y + 6)$

20. $4(y - 1) + 3(y + 2) = 5(y - 4)$

21. $4x - 2(x + 1) = 5(x + 3) + 5$

22. $7 - 2(x - 1) = 3(2x - 1) + 2$

23. $10(2x + 3) - 8(3x - 5) + 5(2x - 8) = 0$

24. $2(x + 4) + 3(x - 10) = 8$

25. $7(2x - 4) + 3(5 - 3x) = 2$

26. $10(x + 4) - 9(x - 3) - 1 = 8(x + 3)$

27. $5(2x - 1) - 2(x - 2) = 7 + 4x$

28. $6(3x - 4) - 10(x - 3) = 10(2x - 3)$

29. $3(x - 3) - 7(2x - 8) - (x - 1) = 0$

30. $5 + 2(x + 5) = 10 - (4 - 5x)$

31. $6x + 30(x - 12) = 2\left(x - 1\frac{1}{2}\right)$

32. $3\left(2x - \frac{2}{3}\right) - 7(x - 1) = 0$

33. $5(x - 1) + 17(x - 2) = 2x + 1$

34. $6(2x - 1) + 9(x + 1) = 8\left(x - 1\frac{1}{4}\right)$

35. $7(x + 4) - 5(x + 3) + (4 - x) = 0$

36. $0 = 9(3x + 7) - 5(x + 2) - (2x - 5)$

37. $10(2.3 - x) - 0.1(5x - 30) = 0$

38. $8\left(2\frac{1}{2}x - \frac{3}{4}\right) - \frac{1}{4}(1 - x) = \frac{1}{2}$

39. $(6 - x) - (x - 5) - (4 - x) = -\frac{x}{2}$

40. $10\left(1 - \frac{x}{10}\right) - (10 - x) - \frac{1}{100}(10 - x) = 0.05$

Example

Solve $(x + 3)^2 = (x + 2)^2 + 3^2$

$$
\begin{aligned}
(x + 3)(x + 3) &= (x + 2)(x + 2) + 9 \\
x^2 + 6x + 9 &= x^2 + 4x + 4 + 9 \\
6x + 9 &= 4x + 13 \\
2x &= 4 \\
x &= 2
\end{aligned}
$$

Exercise 2.3C

Solve:

1. $x^2 + 4 = (x + 1)(x + 3)$

2. $x^2 + 3x = (x + 3)(x + 1)$

3. $(x + 3)(x - 1) = x^2 + 5$

4. $(x + 1)(x + 4) = (x - 7)(x + 6)$

5. $(x - 2)(x + 3) = (x - 7)(x + 7)$

6. $(x - 5)(x + 4) = (x + 7)(x - 6)$

7. $2x^2 + 3x = (2x - 1)(x + 1)$

8. $(2x - 1)(x - 3) = (2x - 3)(x - 1)$

9. $x^2 + (x + 1)^2 = (2x - 1)(x + 4)$

10. $x(2x + 6) = 2(x^2 - 5)$

11. $(x + 1)(x - 3) + (x + 1)^2 = 2x(x - 4)$

12. $(2x + 1)(x - 4) + (x - 2)^2 = 3x(x + 2)$

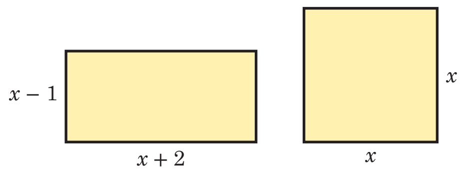
13. $(x + 2)^2 - (x - 3)^2 = 3x - 11$

14. $x(x - 1) = 2(x - 1)(x + 5) - (x - 4)^2$

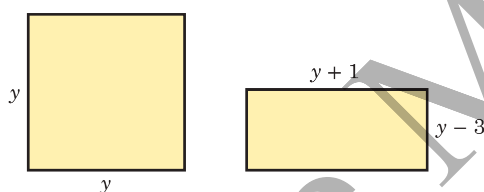
15. $(2x + 1)^2 - 4(x - 3)^2 = 5x + 10$

16. $2(x + 1)^2 - (x - 2)^2 = x(x - 3)$

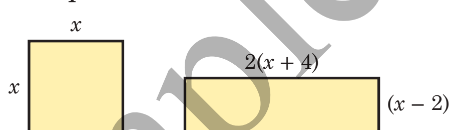
17. The area of the rectangle here exceeds the area of the square by 2 cm^2. Find x.



18. The area of the square exceeds the area of the rectangle by 13 m^2. Find y.



19. The area of the square is half the area of the rectangle. Find x.



Equations involving fractions

When solving equations involving fractions, multiply both sides of the equation by a suitable number or letter to eliminate the fractions.

Example 1

Solve $\dfrac{5}{x} = 2$

$5 = 2x$ (Multiply both sides by x.)

$\dfrac{5}{2} = x$ (Divide both sides by 2.)

Example 2

Solve $\dfrac{x+3}{4} = \dfrac{2x-1}{3}$

$12 \times \dfrac{(x+3)}{4} = 12 \times \dfrac{(2x-1)}{3}$ (Multiply both sides by 12.)

$3(x+3) = 4(2x-1)$ (Or you can cross multiply.)

$3x + 9 = 8x - 4$

$13 = 5x$ (Subtract $3x$, not $8x$, so that the x term is positive.)

$\dfrac{13}{5} = x$

$x = 2\dfrac{3}{5}$

Example 3

Solve $\dfrac{5}{(x-1)} + 2 = 12$

$\dfrac{5}{(x-1)} = 10$ (2 and 12 are like terms so combine them first.)

$5 = 10(x-1)$

$5 = 10x - 10$

$15 = 10x$

$\dfrac{15}{10} = x$

$x = 1\dfrac{1}{2}$

Exercise 2.3D

Solve:

1. $\dfrac{7}{x} = 21$

2. $30 = \dfrac{6}{x}$

3. $\dfrac{5}{x} = 3$

4. $\dfrac{9}{x} = -3$

5. $11 = \dfrac{5}{x}$

6. $-2 = \dfrac{4}{x}$

7. $\dfrac{x}{4} = \dfrac{3}{2}$

8. $\dfrac{x}{3} = \dfrac{5}{4}$

9. $\dfrac{x+1}{3} = \dfrac{x-1}{4}$

10. $\frac{x+3}{2} = \frac{x-4}{5}$

11. $\frac{2x-1}{3} = \frac{x}{2}$

12. $\frac{3x+1}{5} = \frac{2x}{3}$

13. $\frac{8-x}{2} = \frac{2x+2}{5}$

14. $\frac{x+2}{7} = \frac{3x+6}{5}$

15. $\frac{1-x}{2} = \frac{3-x}{3}$

16. $\frac{2}{x-1} = 1$

17. $\frac{x}{3} + \frac{x}{4} = 1$

18. $\frac{x}{3} + \frac{x}{2} = 4$

19. $\frac{x}{2} - \frac{x}{5} = 3$

20. $\frac{x}{3} = 2 + \frac{x}{4}$

21. $\frac{5}{x-1} = \frac{10}{x}$

22. $\frac{12}{2x-3} = 4$

23. $2 = \frac{18}{x+4}$

24. $\frac{5}{x+5} = \frac{15}{x+7}$

25. $\frac{9}{x} = \frac{5}{x-3}$

26. $\frac{4}{x-1} = \frac{10}{3x-1}$

27. $\frac{-7}{x-1} = \frac{14}{5x+2}$

28. $\frac{4}{x+1} = \frac{7}{3x-2}$

29. $\frac{x+1}{2} + \frac{x-1}{3} = \frac{1}{6}$

30. $\frac{1}{3}(x+2) = \frac{1}{5}(3x+2)$

31. $\frac{1}{2}(x-1) - \frac{1}{6}(x+1) = 0$

32. $\frac{1}{4}(x+5) - \frac{2x}{3} = 0$

33. $\frac{4}{x} + 2 = 3$

34. $\frac{6}{x} - 3 = 7$

35. $\frac{9}{x} - 7 = 1$

36. $-2 = 1 + \frac{3}{x}$

37. $4 - \frac{4}{x} = 0$

38. $5 - \frac{6}{x} = -1$

39. $7 - \frac{3}{2x} = 1$

40. $4 + \frac{5}{3x} = -1$

41. $\frac{9}{2x} - 5 = 0$

42. $\frac{x-1}{5} - \frac{x-1}{3} = 0$

43. $\frac{x-1}{4} - \frac{2x-3}{5} = \frac{1}{20}$

44. $\frac{4}{1-x} = \frac{3}{1+x}$

45. $\frac{x+1}{4} - \frac{x}{3} = \frac{1}{12}$

46. $\frac{2x+1}{8} - \frac{x-1}{3} = \frac{5}{24}$

2.4 Problems solved by linear equations

Step 1 Let the unknown quantity be x (or any other letter) and state the units (where appropriate).

Step 2 Express the given statement in the form of an equation. Do not include the units in the equation.

Step 3 Solve the equation for x and give the answer in *words*. (Do not finish by just writing '$x = 3$'.)

Step 4 Check your solution using the initial problem (not your equation).

Example 1

The sum of three consecutive whole numbers is 78. Find the numbers.

Let the smallest number be x; then the other numbers are $(x + 1)$ and $(x + 2)$.

Form an equation:

$x + (x + 1) + (x + 2) = 78$

$3x + 3 = 78$

Solve: $3x = 75$

$x = 25$

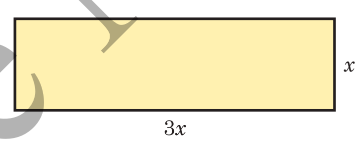
In words:

The three numbers are 25, 26 and 27.

Check: $25 + 26 + 27 = 78$

Example 2

The length of a rectangle is three times its width. If the perimeter is 36 cm, find the width.



Let the width of the rectangle be x cm.

Then the length of the rectangle is $3x$ cm.

Form an equation.

$x + 3x + x + 3x = 36$ or $2(x + 3x) = 36$

Solve: $8x = 36$

$x = \frac{36}{8}$

$x = 4.5$

In words:

The width of the rectangle is 4.5 cm

Check: If width = 4.5 cm

length = 13.5 cm

perimeter = 36 cm

Revision exercise 2

1. Solve these equations.

a) $x + 4 = 3x + 9$

b) $9 - 3a = 1$

2. Given $a = 3$, $b = 4$ and $c = -2$, evaluate:

a) $2a^2 - b$

b) $a(b - c)$

c) $2b^2 - c^2$

3. Solve these simultaneous equations.

a) $3x + 2y = 5$
$2x - y = 8$

b) $2m - n = 6$
$2m + 3n = -6$

c) $3x - 4y = 19$
$x + 6y = 10$

d) $3x - 7y = 11$
$2x - 3y = 4$

4. Given that $x = 4$, $y = 3$, $z = -2$, evaluate:

a) $2x(y + z)$ **b)** $(xy)^2 - z^2$

c) $x^2 + y^2 + z^2$ **d)** $(x + y)(x - z)$

e) $\sqrt{x(1 - 4z)}$ **f)** $\dfrac{xy}{z}$

5. a) Expand and simplify $(x - 2)(x - 3)(x - 4)$.

b) Expand and simplify $(2x - 3)^3$.

6. Solve these equations.

a) $5 - 7x = 4 - 6x$

b) $\dfrac{7}{x} = \dfrac{2}{3}$

7. Find the value of $\dfrac{2x - 3y}{5x + 2y}$ when $x = 2a$ and $y = -a$.

8. Solve these simultaneous equations.

a) $7c + 3d = 29$
$5c - 4d = 33$

b) $2x - 3y = 7$
$2y - 3x = -8$

c) $5x = 3(1 - y)$
$3x + 2y + 1 = 0$

d) $5s + 3t = 16$
$11s + 7t = 34$

9. Solve these equations.

a) $4(2x - 1) - 3(1 - x) = 0$

b) $\dfrac{x + 3}{x} = 2$

10. Given that $m = -2$, $n = 4$, evaluate:

a) $5m + 3n$

b) $5 + 2m - m^2$

c) $m^2 + 2n^2$

d) $(2m + n)(2m - n)$

e) $(n - m)^2$

f) $n - mn - 2m^2$

11. Given that $a + b = 2$ and that $a^2 + b^2 = 6$, show that $2ab = -2$.
Find also the value of $(a - b)^2$.

12. A jar contains 50 US coins, containing a mixture of dimes (10 cents) and quarters (25 cents). The total value of the coins is \$9.35. How many dimes are there?

13. Pat bought 45 stamps, some for 40c and some for 58c. If he spent \$22.50 altogether, how many 40c stamps did he buy?

Examination-style exercise 2

NON-CALCULATOR

1. a) $\frac{5}{7} + \frac{11}{14} = \frac{x}{2}$ Work out the value of x. [1]

b) $\frac{7}{4} \div \frac{4}{y} = \frac{35}{16}$ Work out the value of y. [1]

2. Solve these simultaneous equations. [3]

$4x + y = 17$

$3x - 2y = 10$

3. Solve these equations.

a) $\frac{2x}{3} - 12 = 0$ [2]

b) $\frac{x + 8}{3} = \frac{8x - 1}{11}$ [2]

4. Solve these simultaneous equations. [3]

$0.3x + 2y = 17$

$0.6x + 3y = 27$

5. a) Expand and simplify $(x - 3)^2(3x + 1)$. [3]

b) Fully simplify $(x - 3)^2(3x + 1) - (x + 2)^2$. [3]

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